The role of wage formation in empirical macroeconometric models

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Summary

The specification of model equations for nominal wage setting has important implications for the properties of macroeconometric models that are used to aid economic policy decisions and as tools for forecasting. The modelling of the joint dependencies between wage and price adjustments, and their connections to variables like labour productivity and the rate of unemployment in the economy, is important for explaining inflation and for analyzing how it reacts to for example monetary policy stimulus during and after an economic crisis. Nominal wage rigidity is inherent in modern economies and since wage and price adjustments are imperfectly synchronized, real wage rigidity is an implication. The evolutions of nominal and real wages are therefore system properties that require multiple equation modelling. The importance of nominal wage setting was recognized by the producers of macroeconometric models from the beginning. The intellectual work laid down to resolve the several issues raised by the endogenization wages in macroeconometric models resulted in both the Phillips curve and the equilibrium correction model (later recognized as an implication of co-integrated variables more generally). The main models of wage and price formation are the Phillips Curve (PCM), the wage-price equilibrium correction model (WP-ECM) and the New Keynesian Phillips curve (NKPCM). Both the PCM and the WP-ECM originated in the second half of the 1950s. The PCM was adapted by the producers of macroeconometric models of the 1960s. The WP-ECM began to made its mark on the operational macroeconometric models during the late 1980s, but primarily in European

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models, not in models of the US economy. The NKPCM can be seen as an integral part of the macroeconomic paradigm generically known as Dynamic Stochastic General Equilibrium Models (DSGEs), which became dominant early in the new millennium. The three model classes can be interpreted as different (unique) specifications of the system of stochastic difference equations that defines the supply side of medium term macro econometric models. The econometric treatment of wage formation therefore has implications for the total model properties. This calls for an appraisal of the different wage models, in particular in relation to the concept of the neutral equilibrium rate of unemployment, and to the natural rate of unemployment as a special case thereof. The appraisal also concerns the methods and research strategies used to specify and estimate the wage-price module of a macroeconomic model. The construction of a macroeconomic models used to be a sub-discipline of applied economics that built on the combination of theoretical and practical skills in modelling, including knowledge about institutions. Wage formation relationships in particular were typically viewed as forged between the global forces of markets and national institutions. Under the reign of DSGE models, macroeconomics has become more of a theoretical sub-discipline. Nevertheless, producers of DSGE model make use of hybrid forms if an initial theoretical specification fails to meet a benchmark for acceptable data fit. The NKPCMs found in existing operational DSGE models can be seen as the product of the pragmatic approach. However, an appraisal shows that a common ground exists between the NKPC, WP-ECM and PCM, and that it can used to compare the model types empirically, using a common standard. This may be effective in reviving the objective of empirical macroeconometric modelling: to learn from data while retaining the economic theoretical interpretability of the empirical relationships that represent real world wage formation.

Keywords: Wage and price formation, equilibrium correction, cointegration, Phillips curve, natural rate, NAIRU, collective bargaining, non-natural rate model, New Keynesian Phillips curve, model encompassing, model based forecasts, dynamic properties of models.

The centrality of wage formation in models of real world economies

The specification of the supply side of the economy is important for the properties of a macroeconometric model. The key part of the supply side are represented by those equations that describe the behaviour of firms, in particular price setting and those that reflect the determination of wages, cf. Nickell (1988), Wallis et al. (1984).

In the following, the term wage formation will be used relatively broadly, referring to the process whereby the renumeration of wage earners is determined. Negotiations about wage adjustments and working conditions can take place between individuals and firms, or it can be of a collective nature, which in turn can be at level of the firm, the industry level and in some instances at the national level. In a given historical epoch, national systems of wage formation are mixtures of individual negotiations, collective bargaining and regulation by law. However, the different principles of wage adjustment are usually not equally important.

Hence, collective wage formation refers to systems of national wage formation where collective agreements are negotiated in a majority of industries and sectors and regulate payment and working conditions for a significant part of wage earners. In some countries with collective wage bargaining there are extension mechanisms, implying that collective agreements become extended to potentially large numbers of wage earners who do not hold a union membership. This means that the coverage of collective agreements about wage compensation and working time exceed the degree of unionization in some countries, Nergaard (2014).

Wage formation is an integral part of the processes that drive economic performance. Wage income is the largest component of households' disposable income, which affects aggregated spending and private savings. The wage distribution is important for overall inequality in the economy. Capital income is typically more unequally distributed than wage income. This means that a change in the functional income distribution, from wages to capital, has additional effects on inequalities of modern societies. It is plausible that labour market institutions influence the functional income distribution, but always in combination with the tough realities of global markets forces, Nymoen (2017).

The wage per unit of labour used in the production of goods and services is an important cost factor. Since firms also use labour indirectly, through the purchase of intermediate goods, the share of wages in production costs is larger at the aggregate level that at the level of the individual establishment. Firms deal with increased wage costs in several ways, including mark-up of prices, adaption of labour-saving technologies, outsourcing and other strategies. Sometimes, this has consequences at the macro level. For example a weakening of the trade balance and an increase in the level of unemployment. Yet, as any student of economics will know, a general wage reduction is not necessarily the right solution to an unemployment crisis, because the wage level is also important for activity level through aggregate demand. The effects of wage changes are multiple, and they may go in opposite directions.

Because the general wage level enters into some of the most central processes of modern economies, it follows that the representation of wage formation becomes important in macroeconomic models that are produced to represent real-world economies in simplified form, so they can be analyzed and forecasted.

In the medium-term time perspective, wage formation plays an important role in how shocks to the economy are propagated in the economy. Hence, the decision about how to represent wage formation has implications for the modeling of wage changes over the business cycle, as procyclical or countercyclical.

The propagation of negative shocks to the macroeconomy need not imply a relatively short business cycle, the results might be a stagnation period (long cycle), and the system of wage formation is seen as important for that possibility as well.

In this article one particular focal point is the role of wage formation in the determination of a stationary level of the unemployment rate. The stationary unemployment rate is a broader concept of equilibrium unemployment than the textbook "natural rate" of unemployment, or NAIRU. The wider concept is needed for models with the property that wage inflation can be constant (in the steady-state sense) conditional on a relatively wide set of unemployment rates. Specifically, more factors need to be brought into the picture to determine a unique neutral rate when wage formation is modelled as a system of collective bargaining, than is needed to pin down the natural rate implied by individual negotiations modelled by Phillips curves.

Closely connected to this are policy issues, for example how active the government can be in the provision of monetary and fiscal stimulus without fuelling inflation or contributing to trade balance problems through wage-price and wage-wage spirals. Models of price setting and of wage formation have relevance for these issues, because they imply strategies for feasible good economic performance.

The natural rate of unemployment hypothesis (sometimes just referred to as "u-star", or u^*), was controversial when it was first presented by Friedman (1968) and Phelps (1968), but its acceptance soon spread, first in the United States and then in Europe and in other countries. The natural rate hypothesis has strong policy implications. If inflation is to remain stable, periods during which unemployment is lower than the natural rate must be followed by periods where unemployment is above u^*. Economic policies to regulate activity cannot do more, and indeed should not attempt to do more, than smooth fluctuations around the structural rate of unemployment, u^*, Blanchard (2018).

Although the natural rate hypothesis has been highly successful in influencing monetary policy in particular, it has also been contested on both theoretical and empirical grounds, see e.g., Blanchard and Summers (1986), Hahn and Solow (1997), Cross (1995) and the references therein. It has been argued that the application of u^* models has led to consistent underestimation of slack in USA's labour markets, Galbraith (1997), Solow (1999), as well as in other OECD countries, Storm and Naastepad (2012, Ch. 2).

In the wake of the slow recovery after the international financial crises, and in connection with the stimulus packages to the economies during years of Covid-19, the debate about the slope of the Phillips curve became revitalized. At the Jackson Hole economic policy symposium in 2020, Federal Reserve Chair Jerome Powell spoke about a new medium-term monetary policy strategy, going for maximum employment as opposed to offsetting deviations from assessments of u^*, Powell (2020). This has been interpreted by some as proof that 'the Fed' no longer adheres to the "accelerationist hypothesis", which is a sub-hypothesis of the natural rate hypothesis. The accelerationist hypothesis states that monetary policy cannot sustain unemployment below u^* without leading to higher and higher inflation and in modelling terms it implies a vertical long-run Phillips curve. It remains to be seen whether the position taken by the U.S. Federal Reserve (and other central banks) on the natural rate hypothesis has finally changed. However, the new policy of only reacting to high unemployment, and not to low, is a reminder that beliefs about wage- and price formation have policy relevance. There is a long tradition in economics of giving policymakers' beliefs about how the economy functions a foundation in the econometric evidence and in empirical models, Granger (1990).

The aim of this article is to discuss wage formation in empirical models of the macroeconomy, as opposed to theoretical models. Such model of real world economies come with estimated coefficients and with statistical properties (needed for inference) that depend on how well the specifications of the model equations individually and jointly capture an unknown data generating process (DGP).

In the search of empirical macroeconometric models that are not mis-specified, econometricians make use of both statistical models and economic theories, often in an eclectic way, as neither of these model classes are complete or general, or immediately relevant for real world-data. In the simplest case, where the statistical model is a regression model, the consequence of the position that an unknown DGP has generated the observations is that the regression error term ε_t a derived variable, namely:

$$\underset{\text{observed}}{Y_t} = \underset{\text{explained}}{f(X_t)} + \underset{\text{remainder}}{\varepsilon_t} \tag{1}$$

where Y_t denotes the dependent variable (observed in time period t) that is explained by the use of economic theory and a knowledge of the subject matter. X_t denotes a value of the explanatory variables, and the explanation is given by the function $f(X_t)$. In the regression case, it is the conditional expectation function and it will be the result of a range of decisions, including variable selection and functional form, Hendry (2018). The non-experimental Y_t is not determined or caused by $f(X_t)$, but by a DGP that is unknown, and all variation in Y_t that the explanation does not account for, must therefore "end up" in the remainder ε_t.

Hence, the situation is different from the one the experimental researcher is in, which can be represented as:

$$\underset{\text{result}}{Y_t} = \underset{\text{input}}{g(X_t)} + \underset{\text{shock}}{v_t}, \tag{2}$$

see Hendry (1995a, Ch 1.11). The variable Y_t is in this case interpretable as the result of the experiment, whereas the X_t is the imputed input variable which is decided by the researcher. $g(X_t)$ is a deterministic function. The variable v_t is a shock that leads to some separate variation in Y_t for the chosen X_t.

Again, unlike (2), where v_t represents free and independent variation to Y_t, ε_t in the empirical model (1) is an *implied* variable that gets its properties from the DGP and the explanation $f(X_t)$. Hence, for an empirical econometric model:

$$\varepsilon_t = Y_t - f(X_t) \tag{3}$$

describes that whatever is done on the right-hand side of (3) by changing the specification of $f(X_t)$ or by the measurement of Y_t, the left-hand side is derived as a result, Hendry (1995a, Ch. 2.27).

In the non-experimental situation, the specification of the statistical model of ε_t is conditional on X_t and the functional form. Because the DGP is unknown, the assumed statistical model needs to be tested for mis-specification, see e.g., Spanos (2021), Nymoen (2019, Ch. 2.8). The feasibility of mis-specification testing rests on the properties of the residuals $\hat{\varepsilon_t}$ as valid statistics of the remainder in (1). If any of the assumptions of the statistical model are found to be indefensible after testing, re-specification of the explanation is required (changing the explanatory variables and/or functional form) until statistical adequacy can be demonstrated, at least ideally. In a practical modelling situation, the process of respecification may have to stop before statistical adequacy is complete. In such cases the consequences that any remaining mis-specification has for the estimation of (the parameters of) $f(X_t)$ have to be taken into consideration.

A macroeconomic model that has been completely specified from theory is oddly in analogy with (2), but only by invoking the *axiom of correct specification* (Leamer (1983)), rather than with the empirical model (1). The numerical model output is driven by the asserted input and the assumed probability distribution of the shocks. Hence, the hallmark of an empirical model is that the assertions and assumptions that underlie it are confronted with the data, and revised in the light of any evidence that make them indefensible.

That said, producers of empirical macro models have always made use of economic theory in the model specification process. This is both to give economic interpretability and to clarify the dynamic properties of the model (or in practice, a module of the complete model), see e.g., Bårdsen et al. (2005, Ch. 3). Hence, the difference is not so much whether theory is seen as useful or not for the model specification. The difference is more the attitude toward data confrontation and mis-specification testing, and about taking the consequences in the form of model re-specifications, which in a positive interpretation is learning. At any point in its lifespan, an empirical macro model will always be a result of several compromises.

Lineages of wage modelling in macroeconomics

The representation of wage formation in macroeconometric models has changed over time. During the first epoch, a rather cautious approach to wage formation was taken. The economic theory of the wage variable in the 1950s contained a certain exogeneity proposition about the wage level, Forder (2014, Ch. 1). It was not that the wage level was seen as unconditionally fixed or determined from outside. It was more the point that the wage level was recognized theoretically to be only loosely attached to the level of unemployment. Hence the meaning of exogeneity of nominal wages and prices was with respect to the level of unemployment (and other measures of supply-demand imbalance, like output-gap).

That theoretical view combined two positions: First, that unemployment could change considerably without any noticeable effects on wages and prices at the macro level. This has later become known as the 'L-shaped' wage (and price) curve theory. Second, that wages could change considerably from one period to the next without any simultaneous or preceding change in unemployment. The double insight about wages being endogenous in the economy, while being exogenous with respect to unemployment, was part of a wider recognition that the economic theory of supply and demand could not claim near completeness (or a satisfactory degree of realism) in the area of wage formation. On this several influential theorists can be cited, including Samuelson (1951), Hicks (1955) as well as Ragnar Frisch who in 1945 had singled out wage formation as one the main areas where the economics needed to make progress, both as a scientific and as a practical discipline, Frisch (1945).

However, although empirical macro models began in an epoch when it was understood that economic theory did not imply a clear or complete specification of a wage equation that could be implemented in a macro model, it was also observed that actual wage bargains were struck year after year, and that they were rationalized by considerations of profits, cost of living and relative wages (fairness), Dunlop (1944). This may have encouraged the belief in relatively stable data generating process for wages, that could be put in equation form and included in the macroeconometric models that were developed in the 1950s and 1960s.

It is interesting to note that the idea to relate wage adjustment to a measure of disequilibrium in the labour market happened early in the history of macroeconomic modelling. Klein (1947b) introduced such a relationship into his IS-LM framework (although Klein (1947a) did not; see De Vroy and Malgrange (2010)). The successful Klein-Goldberger (1955) model included an estimated version of that relationship. Unemployment was measured in million workers rather than as a percent, but it nevertheless clearly predated the curve of Phillips (1958). Moreover, Klein and Goldberger's wage equation had the lagged change in the GDP deflator as a second explanatory variable, thus it also foreshadowed what since became known as the augmented Phillips curve model (PCM).

Another real-world development that came to influence the way wages were treated in models, was that inflation in the Western world after the Second World War gradually came to undermine the situation in which relative price stability occurred simultaneously with full employment. By the late 1970s, inflation was seen as a "blight on the stability and efficient performance of the leading economies and to a potential threat to the preservation of democratic societies", Hirsch and Goldthorpe (1978, p. 1). Hence, it is not surprising that during the 1960s and 1970s, the study of economic mechanisms of inflation was intensified in all western countries. With the exception of monetary models of inflation, these studies identified wage setting as an integral part of the inflation process. When combined with specifications of firms' price setting strategies, the approach resulted in wage and price setting equations that could be grafted into a medium-term empirical macro model, to become the supply side of the model, see e.g., Bårdsen et al. (2005). This created an expectation among the users of macroeconometric models that the national wage level was endogenized, and model producers duly responded to the demand. By the mid 1980s it would have appeared old-fashioned to present a macroeconomic model with exogenous rather than endogenous wage level, see e.g., Wallis et al. (1984). Clearly, something had happened along the way.

There is little doubt that the issue about simultaneous achievement of price stability and full employment was one of the factors that gave relevance to the econometric modelling direction known as Phillips curve models, PCMs, This current of the literature may have started with Bill Phillips' 1958 paper, but it soon lost contact with it, as shown by Forder (2014,2019). For example, Lipsey (1960) already reported Phillips curves for the United Kingdom which were not the same across different sample periods, reflecting institutional changes and other structural breaks. Phillips' view had been the opposite, namely that over a long data sample, the relationship that determined the change in money wages was shaped by the forces of supply and demand, as captured by the rate of unemployment, and that institutional factors did not go into it. Hence, Phillips' hypothesis was very different from the consensus theory of the 1950s. If it had been confirmed by Lipsey's work, the incumbent L-shaped wage theory had been proven wrong.

Hence, there is demarcation line between the curve of Phillips from 1958 and the Phillips curves of the literature that came after (Forder's point). However, the name stuck. During the 1960s and 1970s, the standard model became known as the augmented wage Phillips curve because it included multiple explanatory factors besides unemployment. The theoretically most important "augmentation variable" was the expected price change, which gave rise to the distinction between a downward-sloping, short-run PCM, and a long-run curve with a steeper slope, where price expectations are fulfilled. A step in this direction had already been tried with success in the Klein and Goldberger model. In 1972, James Tobin summarized the prevailing view of the dynamics of aggregate supply as consisting of a wage Phillips curve and price mark-up equation (specified in log differences), Tobin (1972). In other words, a PCM. From this era also stems the term natural rate of unemployment, Phelps (1968) and Friedman (1968), and its cousin the NAIRU (non-accelerating inflation rate of unemployment). The natural rate corresponds to the steady-state solution of the system when the long-run Phillips curve is vertical. This also became known as the accelerationist model of inflation. These ideas are very much alive in the macroeconomics of the early 21st century. The slope of the Phillips curve and the role of expectations have been readdressed at irregular intervals, most recently after the Great Recession, cf. Ball and Mazumber (2011,2019) and Blanchard and Summers (2015).

For a young person whose orientation in macroeconomic modelling started in the second decade of the 2000s, when low inflation in the Western economies had become the normal situation, it may be difficult to fully appreciate the formative force that the scientific and practical dimensions of the "price stabilization problem" has had on the development of the field. From this epoch stems the idea that a certain degree of wage coordination could be attained by a system of pattern-wage bargaining known as the Norwegian model of inflation (aka the Scandinavian model), Aukrust (1977), Gjelsvik et al. (2020).[1] As late as in 1990, an issue of the *Oxford Review of Economic Policy* was titled "Inflation" and the first sentence in Stephen Nickell's offering read "Inflation is endemic in Britain", Nickell (1993, p. 26). Nickell, in collaboration with Richard Layard, among others, developed an original model framework for short- and medium-term analysis where the supply side of the economy played a central role. The supply side was represented by a pair of wage and price setting equations, albeit with variables in levels, not in differences as in the PCM. One thesis of the Layard-Nickell model was that the potential of inflation pressure (due to e.g., excessive wage demands by unions) would not necessarily lead to higher inflation. Instead, higher unemployment was the likely outcome, see e.g., Layard and Nickell (1986), Layard et al. (1991) and Blanchard and

[1]This system of coordination has in recent years been favourably reviewed in OECD surveys of Norway: "The system of collective bargaining based on coordinated annual wage increases works well, providing top-level guidance on wage increases that is anchored in macroeconomic realities",OECD (2019, p.37)

Fisher (1989, p. 551-55) among other contributions.

Despite the levels-in-variables formulation of the Layard-Nickell model, it is a natural rate, or NAIRU, model, in the same way as its Phillips curve predecessor. This is due to the "accelerationist" heuristics of the model, Kolsrud and Nymoen (2015). This means that there is a degree of supply-demand balance in the economy, measured by the rate of unemployment, such that wage and price inflation increase if the economy tightens and slow down if the economy is slacker. That special state of the real economy is the natural rate of unemployment or the NAIRU, Solow (1999). It is a unique unemployment rate that reconciles the competing real-wage claims of workers and firms, and it aligns expectations with the steady-state inflation of the model. The implied dynamics of the Layard-Nickell supply-side model therefore belong to the Phillips curve category. Manning (1993) pressed the point by showing that an aggregate wage model equation in levels was under-identified, whereas the associated dynamic wage equation was identified , and then asked (tongue in cheek) the PCM to come back: "all is forgotten".

The category of Phillips curve models has become extended by the New Keynesian Phillips curve (NKPCM), see e.g., Clarida et al. (1999), Svensson (2000), (Woodford, 2003, Ch. 3) among others. There are, however, remarks to this story. In its original derivation and specification, the NKPCM was a model of price inflation with the wage share as a so-called forcing variable, Galí and Gertler (1999). Specifically, this meant that the closed-form solution of the NKPCM was driven by an autoregressive model equation for the wage share. The degree of imbalance in the labour market did not have any role in the model of Galí and Gertler (1999). And it had no implications about nominal wage changes. It appears that the "Phillips curve" part of the name of the new model just followed from the habit among economists to call all model equations normalized on either wage changes or price level changes, for Phillips curves.

However, as the new inflation equation was taken on-board by the producers of New Keynesian dynamic stochastic general equilibrium (DSGE) models, adaptations and modifications have produced NKPCs with properties that are of the accelerationist type. First, the same microeconomic theory that was first used to derive the price NKPC, has been reapplied to wage setting, leading to a wage NKPCM equation, Galí (2011). Second, the users of NKPC models have been pragmatic in the choice of forcing variable. For example, the wage share has been replaced by an output gap variable in the NKPCs used by inflation targeting central banks. Hence, in their reports about model-based forward guidance, inflation is shown to approach its target simultaneously with a closing of the output gap.

New Keynesian Phillips curve models nevertheless stand out because they contain lead-variables (forward-looking) as part of the explanation. This raises challenges for their inclusion in an empirical macro model. However, the section titled The New Keynsian Phillips Curve (NKPCM) covers strategies for empirical assessment of NKPCMs *qua* empirical model equations for supply-side dynamics.

The second main branch of the econometric model development can be seen as a continuation of the L-shaped theory of the 1950s. In these models, it is possible that wage and price growth can be stabilized at a given rate of unemployment. As noted above, this was an important element of the exogeneity theory of wages with respect to unemployment. This branch of wage models goes back to the contributions of Denis Sargan, and they first became known as error-correction models, see Sargan (1964,1971,1980). These models became precursors of formal cointegration analysis, Hendry and Phillips (2019). In current modelling parlance they are referred to as equilibrium correction models, ECMs, because they are representing wages and prices as adjusting toward existing equilibrium relationships, which in turn are interpretable as cointegration relationships (given an $I(1)$ statistical model of the system), Hendry (2003).

When he later revisited his work from 1964, Sargan used what had become commonly known as the Phillips curve as a reference point. He wrote that the error correction form arose "by noting that if the lagged real wage is introduced as a variable into a standard wage Phillips curve it is statistically significant. This is enough to reject models which exclude this variable."[2] Hence, the choice between ECM and Phillips curve versions of the wage equation could be made empirically, using a simple t-test. In practice, it may not be quite that simple because contesting wage model equations may differ in several other respects. However, in most cases a minimum nesting model of the two can be formulated, and the decision can be made either by encompassing testing or by a variable selection procedure.

The economic interpretation of wage-price ECMs has also been developed considerably. The equilibrium correction variable in the wage change equation can be formulated as the difference between last period's nominal wage (in logs) and a wage norm that can be rationalized by theories

[2] Sargan (1971, p. 52), as noted by Ericsson et al. (2001).

of collective bargaining about nominal wages. When combined with a dynamic price setting equation consistent with monopolistic competition among the firms, the ECM wage equation defines a dynamic model of wage and price setting. The model is known by different names. Dynamic incomplete competition model, ICM, is perhaps the most apt name, as it captures that the model is characterized by "deviations from perfect competiton" in both labour and product markets, Bårdsen et al. (2005, Ch. 5). The assumption of monopolistic competition represents a common ground with New Keynesian DSGEs which assumes that the producers of intermediate goods are monopolistically competitive firms (perfect competition is assumed for final goods).

One advantage of wage-price ECMs is that they treat wage-price dynamics as a matter of system dynamics. In these models, it is not possible to conclude about the dynamic stability of nominal wage growth or the real-wage level by looking at the dynamics of each structural equation in turn. System methods are needed. For example the eigenvalues of the system's companion form can in principle be linked to the coefficients of the structural equations. However, in realistic cases the algebra becomes too complex to be of much help for interpretation, Kolsrud and Nymoen (2014). Simulation methods, long favoured by developers of macroeconometric models, therefore remain as the main tool for presentation of dynamic properties of the estimated wage-price system. Both approaches will be used in the discussion of ECM and PCM versions of the wage-price side of stylized macroeconomic models.

A model typology

The different models of the supply side can be represented as members of a model typology, Bårdsen et al. (2005, Ch. 7.5.1). There can be open and closed economy versions of all the models. To simplify notation only the closed economy versions are specified here. Open economy considerations will be introduced in connection with examples given of empirical implementations of the models.

Let w be the wage level and p the price level (GDP deflator or consumer price index); with a as average labour productivity, the wage share is given as $ws = w - a - p$; u is the unemployment rate; all measured in logs. In order to simplify notation, several variables that are often used in macroeconomic models are omitted (e.g., tax rates, foreign prices, and the output gap).

A model of the supply side general enough for the typology then takes the form:

$$\Delta w = \pi_w \Delta p^e + \tau_w \Delta a - \beta_w ws - \sigma_w u, \tag{4}$$

$$\Delta p = \pi_p \Delta p^e + \tau_p (\Delta w - \Delta a) + \beta_p ws, \tag{5}$$

where Δp^e is "expected inflation" and the dynamics is to be specified for each model. Constant terms and random disturbances are omitted for simplicity.

The different models drop out as special cases:

1. The Phillips curve model (PCM):

$$\Delta w_t = \pi_{w1} \Delta p_t - \sigma_{w1} u_t, \tag{6}$$

$$\Delta p_t = \tau_{p1} (\Delta w_t - \Delta a_t). \tag{7}$$

   The first equation is the wage Phillips curve, hence $0 < \pi_{w1} \leq 1$, $\sigma_{w1} > 0$ and (implicitly) $\beta_{w1} = \tau_w = 0$. If $\pi_{w1} = 1$, the wage PCM is vertical, and the natural rate is determined by that relationship alone (zero here since an intercept is omitted).

2. The wage-price equilibrium correction model (WP-ECM):

$$\Delta w_t = \pi_{w2} \Delta p_t + \tau_{w2} \Delta a_t - \beta_{w2} ws_{t-1} - \sigma_{w2} u_t, \tag{8}$$

$$\Delta p_t = \tau_{p2} (\Delta w_t - \Delta a_t) + \beta_{p2} ws_{t-1}. \tag{9}$$

   In equation (9), the equilibrium correction term is simply the lagged wage share, which implies that static or (long-run) homogeneity is imposed.

3. The New Keynesian Phillips curve model (NKPCM) is given as:

$$\Delta w_t = \Delta p_t + \Delta a_t - \beta_{w3} ws_{t-1}, \tag{10}$$

$$\Delta p_t = \pi^f_{p3} \Delta p^e_{t+1} + \pi^b_{p3} \Delta p_{t-1} + \beta_{p3} ws_t, \tag{11}$$

   where the expectations term Δp^e_{t+1} in (11) is to be modelled as a conditional expectation (i.e., the rational expectations hypothesis), see Section . It is customary to refer to the case of

$\pi^b_{p3} = 0$ as the (theoretically) pure New Keynesian Phillips curve, while $\pi^f_{p3} > 0$ and $\pi^b_{p3} > 0$ define the hybrid form of the NKPCM.

One difference between the old and the new Phillips curve model, is that the rate of unemployment appears in the PCM but not in the NKPCM. However, that gap can can be bridged by adapting the theory of staggered nominal adjustment to wages as well as to prices, leading to the New Keynesian *wage* Phillips curve, cf. Erceg et al. (2000), Galí (2011).

The real demarcation line between the old and the new Phillips curve models is that the lead variable Δp^e_{t+1} in the NKPCM is represented by the hypothesis of rational expectations.

The WP-ECM mainly differs from the NKPCM in the treatment of expectations and from the PCM in the latter's exclusion of equilibrium correction mechanisms that follow from the theories of nominal and real wage formation that integrate rent sharing, collective wage bargaining and monopolistic price setting.

It is relatively straightforward to formulate a joint multiple-equation system for the WP-ECM and the PCM. The precise definition of Δp^e_{t+1} as the mathematical conditional expectation of Δp_{t+1} in the NKPCM, makes it necessary to make additional assumptions before the common ground can be enlarged to include that model. We therefore treat WP-ECMs and PCMs in the next section, and return to the NKPCM in the separate section called The New Keynesian Phillips Curve.

WP-ECM and PCM models of the supply side

In this section, an equation system that can represent a model of the supply side of an open economy macro model is analysed. Extension to the open economy does not interfere with the model typology while it is important for the relevance of the framework to operational models. The modelling framework presented has developed over a number of years, and it has been applied to different data sets, see e.g., Kolsrud and Nymoen (1998,2014), Bårdsen and Nymoen (2003,2009ab), Akram and Nymoen (2009), Bårdsen et al. (2012).

Theory

The following presentation follows Nymoen (2017, section 3.3) closely. In order to keep the framework trackable, only two new variables are introduced: the domestic producer price, q, and the import price index denoted in domestic currency, pi. As a consequence, the symbol p can be redefined to be the consumer price index, which represents a weighted average of q and pi.

Nominal and real trends

Following custom an integrated time series is denoted by $I(d)$, where d is the order of integration.[3] The first $I(1)$ variable is the price of imports pi in domestic currency: The generating equation is a Gaussian random walk with a positive drift term, g_{pi}:

$$pi_t = g_{pi} + pi_{t-1} + \varepsilon_{pit},\ g_{pi} > 0 \text{ and } \varepsilon_{pit} \sim N(0, \sigma^2_{pi}). \tag{12}$$

By definition pi_t is the sum of the log of a price index denoted in foreign currency and the log of the nominal exchange rate index. The parameters of (12) therefore depend on the exchange rate regime (fixed or floating). However, the order of integration will be invariant under the assumption that the foreign currency denoted price index is $I(1)$ in both a fixed exchange rate regime and a floating exchange rate regime.

The import price index has a direct impact on the domestic price level through the identity:

$$p_t = \phi q_t + (1 - \phi) pi_t, \tag{13}$$

where p_t is the logarithm of the consumer price index. The coefficient $0 < \phi < 1$ measures the share of imports in total consumption. Although (13) is only a simplification, it serves the purpose of drawing the important distinction between the consumer real wage $(w - p)$ and the producer real wage $(w - q)$.

Returning to the specification of exogenous trends, the logarithm of labour productivity a_t is expressed as:

$$a_t = g_a + a_{t-1} + \varepsilon_{at},\ g_a > 0 \text{ and } \varepsilon_{at} \sim N(0, \sigma^2_a). \tag{14}$$

The specification is chosen for analytical tractability. In a wider setting it is possible to allow for endogenous responses of productivity growth (implying that g_a is a variable not a parameter). Economic theories suggest both a role of demand growth (Kaldor-Verdoorn effect), and of wage

[3] It is understood that any unit-root removed by differencing is a so called zero-frequency root. Specifically, roots a the seasonal frequencies are not considered.

lead technological progress (Schumpeter (1942), Barth et al. (2014)) which may also be regime dependent, see e.g., Storm and Naastepad (2012, Ch. 3) and the references therein.

Real wage norms (targets)

Key to the model formulation is the definition of two different latent real wage variables: The targeted real wage from the firm's own point of view, rw_t^f, and the anticipated real wage, rw_t^b from the setting of the nominal wage level. The two real wages are given by:

$$rw_t^f = w_t - q_t^f = -m_q \qquad + \quad a_t + \vartheta\, u_t, \;\; \vartheta \geq 0 \tag{15}$$

$$rw_t^b \equiv w_t^b - q_t = \quad m_w + \omega\,(p_t - q_t) + \iota\, a_t - \varpi\, u_t, \tag{16}$$

with $\iota > 0$, $0 \leq \omega \leq 1$, $\varpi \geq 0$, see Nymoen and Rødseth (2003), Forslund et al. (2008).

Another term used is "wage norm", which captures the idea of a system where in particular collective bargaining plays a role in defining a norm for the actual wage. In any given point in time, the actual wage level will typically deviate from the norm, but at the same time it also will be attracted by the norm (i.e., equilibrium correction).

In the price setting equation (15), q_t^f denotes the price level set by the firm on the basis of expected nominal marginal labour costs $w_t - a_t$.[4] The case of $\vartheta = 0$, is known as normal cost pricing. This hypothesis states that any procyclical fluctuations in the mark-up of prices over actual unit costs are merely side effects of fluctuations in productivity, cf. Barker and Peterson (1987, Ch. 13.5).

In (16), w_t^b represents the wage compromise reached by the parties. Different theories of wage formation all imply that the elasticity ι with respect to productivity is positive. In systems with strong collective bargaining, $\iota = 1$ has both theoretical and empirical support, Forslund et al. (2008). The standard assumption about the sign of the coefficient for unemployment, ϖ, is that it is non-negative, hence $-\varpi < 0$, as indicated. ω is known as the wedge coefficient since it is multiplied by $(p - q)_t$ which is the difference (or wedge) between the consumer and the producer real wage.[5] The

[4] To simplify notation, firms' expected wage and wage earners' anticipations about cost of living, are implicit in the model. Due to zero mean $I(0)$ expectation errors, these variables will not have any implications for co-integration (or not) between the variables. However, it is understood that w_t in (15) is an expected variable, and that p_t and q_t in (16) likewise denote expected prices in this context.

[5] Tax rates are abstracted from in this presentation.

wedge coefficient is assumed to be non-negative, $\omega \geq 0$, see Rødseth (2000, Ch. 8.5).

In presentations of the Layard-Nickell model mentioned above, equations that are almost the same as (15) and (16) are motivated by the economic theory of price setting by firms and by formal wage bargaining models. However, in the Layard-Nickell version, it is assumed that $rw_t^f = rw_t^b = rw_t$. If it is further assumed that the wedge variable $p_t - q_t$ is fixed from outside the wage-price setting model, usually with reference to requirement of external balance, (15) and (16) determine the real wage and the rate of unemployment as endogenous variables. In this interpretation, increased pressure for higher nominal wages (higher m_w) leads to higher unemployment, not to higher inflation (which is assumed to be constant under the assumption of $rw_t^f = rw_t^b$), which was a main point in Nickell's article about inflation being endemic in the United Kingdom.

Returning to the intended interpretation of (15) and (16), which is that rw_t^f and rw_t^b are $I(1)$ variables that may be jointly cointegrated with rw_t, define two variables ecm_t^f and ecm_t^b:

$$ecm_t^f \equiv rw_t - rw_t^f = q_t^f - q = w_t - q_t - a_t - \vartheta\, u_t + m_q \tag{17}$$

$$ecm_t^b \equiv rw_t - rw_t^b = w_t - w_t^b = w_t - q_t - \iota\, a_t - \omega(1-\phi)re_t + \varpi\, u_t - m_w, \tag{18}$$

where (13) has been used to rewrite the wedge term:

$$(p-q)_t = (1-\phi)(pi-q)_t,$$

and then define the *real exchange rate* $re_t \equiv pi_t - q_t$. If the hypothesis about cointegration is tenable, both ecm_t^f and ecm_t^b are $I(0)$ variables. But it is not necessary to impose the cointegration from the outset.

Wage-price dynamics

To set wage-price dynamics in model form it is convenient to use the simultaneous equations model (SEM):

$$\Delta q_t = c_q + \psi_{qw}\,\Delta w_t + \psi_{qpi}\,\Delta pi_t - \varsigma\, u_{t-1} + \theta_q ecm_{t-1}^f + \varepsilon_{qt}, \tag{19}$$

$$\Delta w_t = c_w + \psi_{wq}\,\Delta q_t + \psi_{wp}\,\Delta p_t - \varphi\, u_{t-1} - \theta_w ecm_{t-1}^b + \varepsilon_{wt}, \tag{20}$$

where Δ is the difference operator, $\Delta q_t \equiv q_t - q_{t-1}$, and where ψ_{qw}, ψ_{qpi}, ψ_{wq}, ψ_{wp}, ς, φ, θ_q, $\theta_w \geq 0$, $\varepsilon_{qt} \sim N(0, \sigma_q^2), \varepsilon_{wt} \sim N(0, \sigma_w^2)$.[6]

The inclusion of u_{t-1} in both (19)and(20) may seem unnecessary as it appears in ecm_t^f and ecm_t^b from before. However, this is to have a framework that encompasses the case where cointegration fails and ecm_t^f and/or ecm_t^b are $I(1)$ instead of $I(0)$. In wage setting, $ecm_t^b \sim I(1)$ would imply $\theta_w = 0$, but in this case $\varphi > 0$ is a possibility as long as $u_t \sim I(0)$, and a wage Phillips curve model is then implied. Conversely, $ecm_t^b \sim I(0)$ implies $\theta_w > 0$ by the Granger-Engle (1987) representation theorem. And in this case $\varphi = 0$ is the only logically consistent possibility.

Using equations (17) and (18) to substitute ecm_{t-1}^f and ecm_{t-1}^b the SEM for wage-price dynamics becomes:

$$\begin{aligned} \Delta q_t = {} & (c_q + \theta_q\, m_q) + \psi_{qw}\, \Delta w_t + \psi_{qpi}\, \Delta pi_t - \mu_q\, u_{t-1} \\ & + \theta_q\, (w_{t-1} - q_{t-1} - a_{t-1}) + \varepsilon_{q,t}, \end{aligned} \tag{21}$$

$$\begin{aligned} \Delta w_t = {} & (c_w + \theta_w\, m_w) + \psi_{wq}\, \Delta q_t + \psi_{wp}\, \Delta p_t - \mu_w\, u_{t-1} \\ & - \theta_w\, (w_{t-1} - q_{t-1} - \iota\, a_{t-1}) + \theta_w\, \omega\, (p_{t-1} - q_{t-1}) + \varepsilon_{w,t}. \end{aligned} \tag{22}$$

noting that the the use of the notation $\mu_q = \theta_q\, \vartheta + \varsigma$ and $\mu_w = \theta_w\, \varpi + \varphi$ makes is possible to encompass the PCM and the wage-price ECM specification.

As noted above, equilibrium correction, and its forerunner error correction, stem from dynamic econometrics. Sargan (1971, p 52) saw "error correction" variables as relevant in a model of bilateral monopoly. He also stated in a later paper that "clearly both sides in a wage bargaining procedure are concerned with the real wage", see Sargan (1980, p 98). In the notation here, this is consistent with $\theta_w > 0$. Later theoretical developments, show formally that $\theta_w > 0$ is implied by collective bargaining, see e.g., Forslund et al. (2008).

The recognition that nominal wage setting was a result of bargaining with real power relationships that reflected institutions, used to be quite common. Another econometrician, Trygve Haavelmo, presented the same line of thought in the mid 1970s, see Anundsen et al. (2012). The label 'conflict models' was sometimes put on these models at the time, but a more apt name might

[6]For coefficients ψ_{wq}, ψ_{qw} and ψ_{wp}, ψ_{qpi}, the non-negative signs are standard assumptions. Negative values of θ_w and θ_q, can give rise to explosive dynamics in wages and prices (hyperinflation), which is different from the low to moderately high inflation scenario that is modelled here.

have been "Compromise models". If an equilibrium real wage existed in Haavelmo's dynamic model, it represented a compromise, like a "quasi peace".

The alternative model to the WP-ECM is the wage Phillips curve, defined by $\theta_w = 0$, Bårdsen et al. (2005, Ch.3-6). Symmetrically, the price Phillips curve is implied by setting $\theta_q = 0$ in out model. In sum, to distinguish between WP-ECM and PCM specifications set:

$$\text{WP-ECM: } \theta_w,\, \theta_q \;> 0 \text{ and } \varsigma = \varphi = 0 \Rightarrow \mu_w = \theta_w \varpi \text{ and } \mu_q = \theta_q \vartheta$$

$$\text{PCM: } \theta_w = \theta_q = 0 \text{ and } \varsigma = \varphi > 0 \Rightarrow \mu_w = \varphi \text{ and } \mu_q = \varsigma$$

It is quite possible to have "mixed cases", for example a wage Phillips curve together with a price ECM, as a reflection of imperfect competition on product markets.

Real-wage dynamics

Equations (21) and (22) in combination with the differenced version of (13):

$$\Delta p_t = \phi\, \Delta q_t + (1 - \phi)\, \Delta pi_t \tag{23}$$

allow the model to be written as a VAR in the two variables re_t and $ws_t = w_t - q_t - a_t$, the logarithm of the wage share. If the parameter restriction $\iota = 1$ on a_{t-1} (in wage-setting) is imposed, the VAR of re_t, ws_t becomes independent of the labour productivity level. Because a_t has been specified as a random walk with drift, this step is to save notation. The analysis of conditions for dynamic stability will be unchanged if the model is specified with a productivity corrected producer real wage: $rwc_t = w_t - q_t - \iota a_t$. This is useful for the analysis of empirical models where $\iota = 1$ cannot be imposed (an example for U.S. data is given below).

The system with re_t, ws_t as endogenous is conditional on u_{t-1} (and Δpi_t and Δa_t). The system is of interest because it can be used to answer one of the classical questions in this field, namely whether it is logically possible for wage-price dynamics to be stable for a given rate of unemployment, not only for a natural rate (i.e.,u^*). This question was analyzed by Kolsrud and Nymoen (1998) who formulated a set of sufficient conditions for (global asymptotical) stability:

$$\theta_w > 0 \quad \text{and} \quad \theta_q > 0 \quad \text{and} \quad \omega > 0 \quad \text{and} < 1 \quad \varphi_{qw} < 1. \tag{24}$$

The first two conditions represent equilibrium correction of wages and prices with respect to their targets. The third condition states that there is a long-run wedge effect in nominal wage-setting. Finally a particular form of dynamic homogeneity is precluded by the fourth condition: for stability a one point increase in the rate of wage growth must lead to less than one point increase in the rate of price growth. Note that $\varphi_{qw} = 1$ is different from dynamic homogeneity in general which would entail:

$$\psi_{qw} + \psi_{qpi} = 1 \tag{25}$$

$$\psi_{wq} + \psi_{wp} = 1. \tag{26}$$

Dynamic homogeneity, in this usual sense, is consistent with a stable steady state of the WP-ECM at a given rate of unemployment. Hence, every unemployment rate is an equilibrium rate, consistent with dynamic stability of the wage-setting process. It is an interesting result, since in the PCM, dynamic homogeneity is exactly what defines a vertical Phillips curve and hence a unique u^*.

In econometric terms, the above is based on the assumption about strong exogeneity of u_t, i.e., no feedback from the real wage of the real exchange rate on the unemployment rate. In the context of modelling the total economy, this is unsatisfactory. In order to allow for joint feedback effects, the following (closed) VAR with three endogenous variables is specified:

$$\underbrace{\begin{pmatrix} re_t \\ ws_t \\ u_t \end{pmatrix}}_{\mathbf{y}_t} = \underbrace{\begin{pmatrix} l & -k & n \\ \lambda & \kappa & -\eta \\ -\rho & \varrho & \alpha \end{pmatrix}}_{\mathbf{R}} \underbrace{\begin{pmatrix} re_{t-1} \\ ws_{t-1} \\ u_{t-1} \end{pmatrix}}_{\mathbf{y}_{t-1}} + \underbrace{\begin{pmatrix} e & 0 & -d \\ \xi & -1 & \delta \\ 0 & 0 & c_u \end{pmatrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} \Delta pi_t \\ \Delta a_t \\ 1 \end{pmatrix}}_{\mathbf{x}_t} + \underbrace{\begin{pmatrix} \epsilon_{re,t} \\ \epsilon_{ws,t} \\ \epsilon_{u,t} \end{pmatrix}}_{\epsilon_t} \tag{27}$$

The third row of (27) contains a simple dynamic relationship between u_t and the lagged re_t and ws_t. It is interpretable as a stylized dynamic aggregate demand relationship. Higher re_t represents real depreciation and hence it is reasonable that the coefficient ρ is non-negative, $\rho \geqq 0$. The assumption $0 < \alpha < 1$ can be said to "contribute" to stationary unemployment. However, $u_t \sim I(0)$ is not implied by it, as stationarity is a system property. The error-term $\epsilon_{u,t}$ contains all other variables that affect u_t.

For the WP-ECM, the **R** and **P** coefficients associated with re_t can be shown to be, Kolsrud and Nymoen (2014))

$$
\begin{aligned}
l &= 1 - \theta_w \, \omega \, \psi_{qw} \, (1-\phi)/\chi, \\
k &= (\theta_q - \theta_w \psi_{qw})/\chi, \\
n &= (\mu_q \, + \mu_w \, \psi_{qw})/\chi, \\
e &= 1 - (\psi_{qpi} + \psi_{qw} \, \psi_{wp} \, (1-\phi))/\chi, \quad = 0 \text{ if dynamic homogeneity} \\
d &= (m_q \, \theta_q + c_q + (m_w \, \theta_w + c_w) \, \psi_{qw})/\chi,
\end{aligned}
$$

where the denominator is: $\chi = 1 - \psi_{qw}(\phi\psi_{wp} + \psi_{wq}) > 0$. The coefficients of ws_t in $\mathbf{R}$ and $\mathbf{P}$, in the case of WP-ECM are:

$$
\begin{aligned}
\lambda &= \theta_w \, \omega \, (1-\psi_{qw})(1-\phi)/\chi, \\
\kappa &= 1 - (\theta_w \, (1-\psi_{qw}) + \theta_q \, (1-\psi_{wq} - \phi \, \psi_{wp}))/\chi, \\
\eta &= (\mu_w \, (1-\psi_{qw}) - \mu_q \, (1-\psi_{wq} - \phi \, \psi_{wp}))/\chi, \\
\xi &= (\psi_{wp} \, (1-\psi_{qw})(1-\phi) - \psi_{qpi} \, (1-\psi_{wq} - \phi \, \psi_{wp}))/\chi, \qquad = 0 \text{ if dynamic homogeneity} \\
\delta &= ((m_w \, \theta_w + c_w)(1-\psi_{qw}) - (m_q \, \theta_q + c_q)(1-\psi_{wq} - \phi \, \psi_{wp}))/\chi.
\end{aligned}
$$

With the exception of δ, which can be both positive and negative, the coefficients are non-negative for reasonable values of the structural parameters, see Kolsrud and Nymoen (2014).

Stability and steady-state rate of unemployment

As is well known, the condition for global asymptotic stability is that the three eigenvalues of $\mathbf{R}$ have modulus inside the unit-circle. However, the general analytic expressions for the eigenvalues of $\mathbf{R}$ are too large and complex to be of much help. Hence, already in this case, where wage formation is attached to the simplest possible representation of the demand side of the economy, simulation of a numerical version of the theoretical model is needed to give better understanding of the system's dynamic properties.

However, one result that can be established from studying the expressions for the eigenvalues, is that the dynamic homogeneity restrictions (25) and (26) have no direct implication for dynamic stability. Moreover, if the system *is* stable, those restriction imply that the steady states of re_t, ws_t

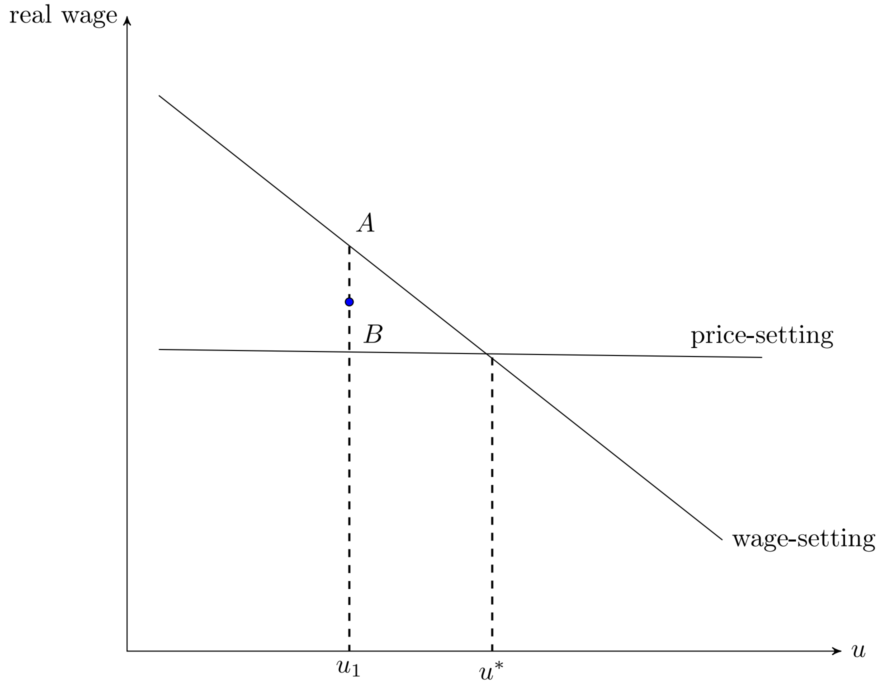

Figure 1: Real wage and unemployment determination. NAIRU (u^*) in the figure and another steady-state rate of unemployment u_1

and u_t do not depend on the nominal growth rate. Hence, there is no long-term effect of currency rate depreciation in this model.

$\theta_w = \theta_q = 0$, defines the PCM version of the system. $\mathbf{R}$ then becomes:

$$\mathbf{R}_{PCM} = \begin{pmatrix} 1 & 0 & n \\ 0 & 1 & -\eta \\ -\rho & \varrho & \alpha \end{pmatrix}$$

which is seen to have a unit-root. Hence, the PCM is less "dynamically stable" than the WP-ECM. Intuitively, because in the PCM, the price and wage growth equations have become disconnected from the lagged real wage and the lagged real exchange rate, other mechanisms must secure the stabilization. However, that may not always be possible. Hence, as $(R - I)$ has rank 2 in the PCM case, there will in general be stochastic trends in all three endogenous variables. Nevertheless, there are two implied cointegration relationships between the I(1) variables.

Figure 1, with the lines representing the "wage curve" and the "price curve" mentioned, represents some of the possibilities. In the model, the case of a singular steady-state value of unemployment (the

u^*) consistent with constant inflation may be specific special case. It is depicted by u^* in the figure, and it illustrates the main message in the Layard-Nickell model, namely that it is unemployment that secures coordination and consistency between conflicting real wage targets:

> 'Only if the real wage (W/P) desired by wage-setters is the same as that desired by price setters will inflation be stable. *And, the variable that brings about this consistency is the level of unemployment'.*[7]

However, in the dynamic model there are other possible solutions. The NAIRU u^* is given by the intersection of the curves, but another steady-state rate of unemployment u_1 may be lower than u^*, which is the case shown in the figure, or may be higher. The figure further indicates (by a •) an equilibrium wage share at a point on the line segment A-B: Heuristically, this is a point where price setters are trying to attain a lower real wage by nominal price increases, at the same time as the wage bargain is delivering nominal wage increases that push the real wage upward, it is a tug-of-war equilibrium.

Hence, fixing the rate of unemployment is not enough to determine the real wage level, and (within the linear in parameter model) any fixed rate of unemployment can be consistent with constant wage inflation in steady state. Interestingly, another example of the non-unique NAIRU is found in the macroeconomic model developed by Hahn and Solow (1997). Their model is different from the one used here (it has sophisticated micro foundations), but the implication that there is no single critical (or natural) level of unemployment at which supply-side dynamics is stabilized, is the same. Even further removed from our model is the famous Diamond-Mortensen-Pissarides (DMP) search and matching models used in academic economics, see e.g., Diamond (1982) and Mortensen and Pissarides (2011). Nevertheless, the indeterminacy of wages also characterizes DMP-type models where the equilibrium wage is only set determined, Hall (2005). In order to determine the level of the equilibrium wage, more theory about wage adjustment has to be added to the core search model with a Nash bargaining solution, for example a degree of real wage rigidity, Krogh (2016).

Returning to Figure 1, another interpretation is that u^* represents an initial steady-state situation (where the economy has been "at rest" for some time), and u_1 represents a new predetermined

[7] Layard et al. (1994, p 18), authors' italics.

steady state after a shock. In this interpretation, there must be a dynamic process that connects the two steady states. One possibility is that the wage setting curve drifts away from its initial position, finally reaching its new stationary position, close to B, after an adjustment period, so that u_1 becomes the new steady-state rate of unemployment (or NAIRU if one sticks to that terminology).

The model dependency of the equilibrium (steady-state) rate of unemployment can also be shown formally in the model framework herer, as special cases that are known from the literature.

For example, if the demand side is simplified by setting $\rho = 0$, meaning that unemployment only depends on the real wage, the wage Phillips curve version of the model drops out as a special case by imposing the restrictions $\theta_w = 0$ (nominal wage setting) and $\theta_q = 0$, $\varsigma = 0$ (price adjustment). If the system is dynamically stable, the steady state rate of unemployment becomes:[8]

$$u^{\text{WPCM}} = \frac{c_w - g_a}{\varphi} + \frac{(\psi_{wq} + \psi_{wp} - 1)}{\varphi} g_{pi}. \tag{28}$$

Subject to a dynamic homogeneity restriction: $\psi_{wq} + \psi_{wp} = 1$, the steady-state rate is independent of "imported inflation" g_{pi}:

$$u^{\text{WPCMr}} = \frac{c_w - g_a}{\varphi}, \tag{29}$$

and defines a natural rate of unemployment within the above framework. Even though this may be the expression that most economists would write down when asked, it is nevertheless a special case. It is not natural in the sense of being a common steady-state value to expect from a dynamics wage price system that allows for more direct equilibrium correction than the adjustment that "goes through" unemployment. It is of considerable interest, as a specifically *restricted* form of Phillips curve dynamics associated with a vertical long-run Phillips curve, central to the accelerationist theory.

If instead $\theta_w > 0$ is assumed, but retaining the other assumptions that led to (29), the expression for the steady-state rate of unemployment becomes:

$$u^{\text{WECM}} = \frac{\theta_w c_u - g_a + (\psi_{wq} + \psi_{wp} - 1) g_{pi}}{\theta_w (1 - \rho) + \varpi \varrho}, \tag{30}$$

[8] Under these assumptions the system separates into a stationary subsystem of u_t and ws_t and a separate random walk process for re_t.

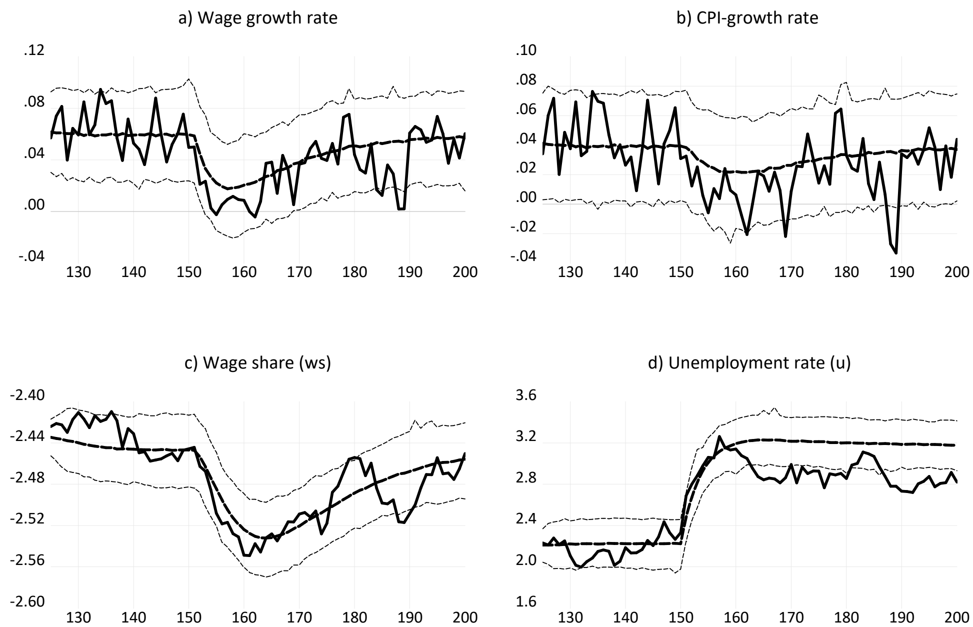

Figure 2: Solution paths for endogenous variables shown in graphs with dashed lines, together with 'actuals' (the generated time series) and 95 percent uncertainty intervals (dotted lines). The first solution period is period 125 and the final period of the dynamic simulation is number 200.

and the superscript WECM indicates that in this special case it is only wage formation that equilibrium corrects around the target (hence $\theta_q = 0$ as noted).

Clearly, the two steady states will be different in general. It may be noted in particular that u^{WECM} depends on c_u which is a parameter from the demand side of the model. A structural change in that parameter will affect u^{WECM} whereas u^{WPCM} is invariant to such a structural break.

Dynamic simulation of the theory model

To demonstrate the dynamic properties of the system, when it is stable, a calibrated version of the WP-ECM can be simulated. For example, a calibration with $\theta_w = 0.15$ and $\theta_q = 0.12$ captures the essence of the system, whereby the real wage targets serve as equilibrium correction mechanisms for nominal wage and price changes. Consistent with this, $\varsigma = \varphi = 0$ can be set. Finally, the model has been calibrated with a large structural break in period 150, to simulate a permanent change in the mean of u. In the experiment, the time series generated by the calibrated version of the VAR (27) are used to estimate the SEM version of the wage-price model by the use of full information maximum likelihood. After estimation, the SEM was simulated dynamically, and the solutions paths of the selected variables were plotted in Figure 2.

Panels a) and b) of Figure 2 plot wage and price inflation, which are quite stable early in the period. The reductions that appear in period 151 are caused by the structural break in the rate of unemployment in period 150. Because that shift is permanent, and the model is non-accelerationist, the plot of the rate of unemployment in panel d) does not return to the initial low level.

Panel c) in Figure 2 shows the wage share. Without the break in u_t, this variable would be stationary, as indicated by the solutions for the period leading up to the increase in unemployment. After the break, the wage share, slowly equilibrium corrects back to the pre-break mean, after first being markedly reduced as a consequence of the step increase in u_t

Hence, although the model assumes that the workers (or their unions) have influence over the money-wage bargain, the simulations confirm that they do not control the dynamics and the long-run level of the wage share. If that was the case, the wage share would be seen to be reduced by the step-up in the unemployment rate.

The insight that the real wage may be only weakly linked to the nominal wage setting is an old one in macroeconomics. Keynes himself was clear about this, see Keynes (1936, p. 12-13). In terms of econometric concepts, the dynamics shown in the figure demonstrates endogenous *co-breaking*, Clements and Hendry (1999, Ch. 9): There are "permanent large-shifts" in the real exchange rate and in unemployment, but there is no break in the unconditional mean of the equally endogenous wage share, even if the nominal wage in particular is directly affected by unemployment.

Returning to panels a) and b), note that the nominal growth rates are stable (and the same) on each side of the structural break in the unemployment rate. Clearly, in this simulation, a unique steady-state rate of unemployment does not follow from constant wage and price inflation rates, as it will do in a PCM of the natural rate (u-star) type. The economic intuition does not lie with L-shaped wage and price curves, but with the equilibrium correction dynamics of the nominal changes with the respect to the lagged wage share.

The simulation results would have been symmetrical with a higher initial steady-state unemployment rate, which then went over a cliff in period 150. Hence, although the model has several features that seem quite acceptable to mainstream economics, it has one implication that stands out sharply from standard macroeconomic theory: The belief that deviations from a natural rate of unemployment toward lower unemployment will create increasing inflation, with the implication that all ambitions to promote near full employment are doomed to fail. Taken at face value, the implication is more in line with the revised monetary policy strategy announced by the U.S. Fed in 2020, namely to "go for" full employment before dismantling the monetary stimulus packages. Note however, that the new monetary policy strategy appears to be based on the belief that the U.S. PCM has flattened. In comparison, no assumption about a pervasive structural break in wage formation was made in the WP-ECM model, where the non-accelerationist property is inherent.

However, care must be taken, as by only looking at one specific calibration, the complex dynamics that the framework can generate can be underestimated. For example, Kolsrud and Nymoen (2014) showed that the interplay between parameters can give rise to very different dynamics, some with cycles, others with more smooth stabilization after a shock. This is also echoing Sargan (1980, p 108), who noted that the dynamics of his wage and price model was "critically dependent" on the estimated coefficients.

Implications for impulse responses, dynamic multipliers and forecasts

The result that the dynamic solution and the eventual steady-state of WP-ECMs and PCMs of the supply side are different, has consequences for other model properties.

Impulse response functions have the same properties as the homogenous part of the solution equations for the endogenous variables. Hence, if the solutions are model dependent, the impulse responses will also be different between models.

Dynamic multipliers (derivatives with respect to non-modelled observed variables) become dominated by the homogenous solution after the impact multiplier and a first couple of dynamic multipliers. Hence, in principle, the dynamic multipliers will also be different between models, although by how much is an empirical question that also depends on the specification of the other parts of the model, not only the wage equations (the phenomenon of extended equilibrium correction).

Finally, model-based dynamic forecasts also become dominated by the homogenous solution (a few periods into the forecast horizon, hence they will be different for WP-ECM and PCM models of the supply side.

The following paragraphs give examples of empirical version of the supply side framework to data sets from different countries and epochs.

Empirical illustrations and applications

US wage-price dynamics

Bårdsen and Nymoen (2009b) specified and estimated a PCM for annual U.S. data (1962-2004). This type of model is the standard North American model of the natural rate of unemployment, following Blanchard and Katz (1999) who argued that a wage equation on equilibrium correction form changes the dynamics fundamentally and can be seen as typical of Europe.

However, exactly how consequential the difference in nominal wage formation becomes for the total model properties depends on the strength of equilibrium correction elsewhere in system. This phenomenon was dubbed *extended equilibrium correction* by Bårdsen and Nymoen (2009b), and shows that the issue about mean reverting behaviour of the rate of unemployment can involve equilibrium correction elsewhere in the system, not only in the nominal wage equation.

The PCM specified by Bårdsen and Nymoen (2009b) had the restrictions $\theta_q = \theta_w = 0$ imposed, whereas the two coefficients of unemployment in the wage and price Phillips curves (ς and φ) are freely estimated. They then specified an ECM model, with θ_w freely estimated, but where the price equation was identical to the PCM version (hence $\theta_q = 0$).

In the specification of the W-ECM they also discovered empirically that $\omega = 1$ in the wage-target equation. This implies that the real wage that defines the wage equilibrium correction term is the consumer real wage $w - p$. They also found it indefensible statistically to impose the restriction $\iota = 1$ on the productivity coefficient, and estimated it to be less than 0.5 (but still significantly different from zero).

In the unemployment equation, both the producer real wage (with coefficient ϱ) and the real exchange rate (with coefficient ρ) entered significantly. The simulated solutions of the empirical W-ECM and the PCM turned out to be nearly identical. However, the paradox can be resolved by taking into account the phenomenon of extended equilibrium correction, which clearly dominated the effects of the different specifications of the wage equation.

In order to specify a PCM that behaved distinctively differently from the W-ECM, and more in line with the textbook case of a vertical long-run Phillips curve, Bårdsen and Nymoen considered a *restricted* econometric PCM where there is no extended equilibrium correction. Figure 3 shows how the three different models respond to a permanent and exogenous shock to unemployment,

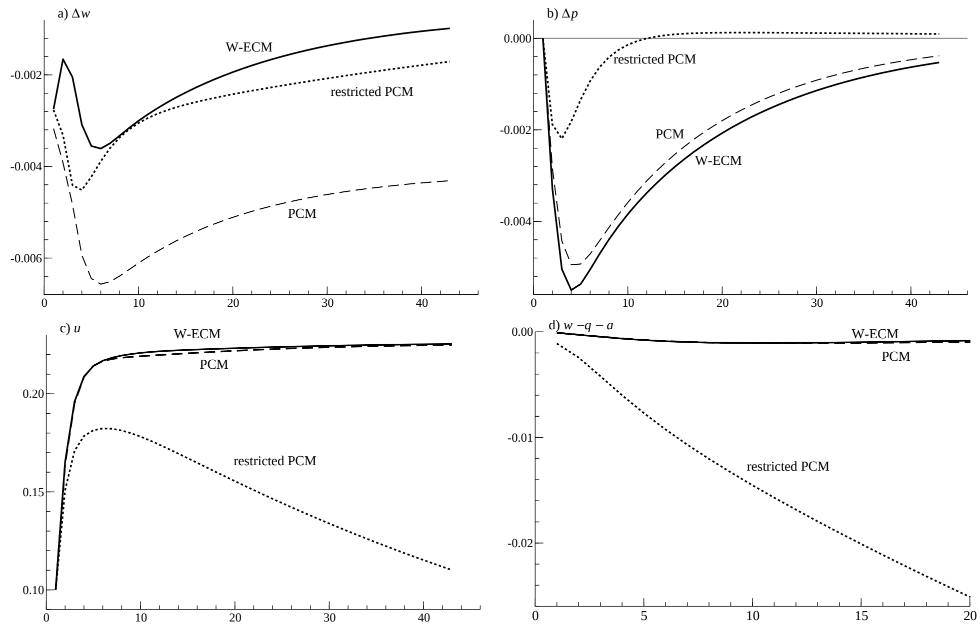

Figure 3: Dynamic multipliers of the econometric models PCM, W-ECM, and the restricted PCM, to a permanent exogenous 0.5 reduction in the unemployment percentage.

specifically to a reduction in the parameter c_u in the third row of the VAR. The shock has been calibrated to correspond to a reduction from 5% to 4.5% in the unemployment rate. In Figure 3, panel a), the graph for the econometric PCM shows the most vigorous wage response, corresponding to a lowering of the annual rate from 5% to 4.4% in the third year after the shock. There is less marked difference between the responses of the W-ECM and the PCM in panel b), which shows the inflation response, which is due to the direct effect of the rate of unemployment in the Δp_t equation of both models.

The differences between the three models are also apparent in panels c) and d), showing the cumulated multipliers for unemployment and in the wage share. For the W-ECM and the PCM, there is a sharp and lasting increase in the rate of unemployment. This kind of response cannot be reconciled with the stylized Phillips curve model, which only allows shocks that arise in the Phillips curve equation to affect the steady-state unemployment rate. Nevertheless, the responses in Figure 3 happen for perfectly logical reasons since the empirical W-ECM and PCM are in fact quite similar in this case, showing the force of extended equilibrium correction.

The graph for the restricted econometric PCM in panel c) shows the response pattern that corresponds to the accelerationist Phillips curve model and to the theoretical PCM steady-state unemployment rate in equation (28). Since the steady-state unemployment rate of this specification of the model only depends on the parameters of the wage Phillips curve (it is a u^* model) the shock to the unemployment rate has to be reversed completely before a new equilibrium can be restored. The single equilibrating mechanism of the model is the response of Δu_t to the lagged wage share, which therefore has to be brought down to a new steady-state level. As can be seen in panels c) and d), the speed of adjustment is very low. For practical purposes it is as if the level of unemployment never returns to its initial and natural value. Thus, in the *restricted* PCM, corresponding to the standard natural rate model, the single equilibrating mechanism is very weak, making the stationarity of u_t more of a formality than an important feature of the dynamic system.

Bårdsen and Nymoen (2009b) showed that the set of hypotheses that separates the *restricted* PCM from the PCM was statistically rejected. Hence the natural rate hypothesis was rejected in their analysis of the U.S. data.

After the Great Recession, influential economists have again asked whether the natural rate hypothesis should be rejected. The evidence considered in this round of "NAIRU-reconsideration" came from both from macro and micro data, but did not lead to a clear-cut conclusion as policy makers were advised to "keep the natural rate hypothesis as their null hypothesis, but keep an open mind and put some weights on the alternatives" (Blanchard (2018). One lasting result may yet prove to be a certain reduction in the belief in the near vertical Phillips curve, Ball and Mazumder (2011), Blanchard (2016b), Hazel et al. (2020). It is plausible that these changes in beliefs about the "US Phillips Curve" played a role in the thinking that led to the revised monetary policy strategy of the U.S. Federal Reserve that was announced in August 2020.

Wage-price specifications and optimal monetary policy

As noted above, the accelerationist Phillips curve has strong implications for monetary policy, and can be recast as a hypothesis about potential economic activity being independent of monetary policy shocks, Blanchard (2016a). However, because of extended equilibrium correction and, realistically, a not completely accelerationist curve, the consequences of Phillips curve specification are more about different sizes and persistence of effects than about complete independence.

Akram and Nymoen (2009) showed how optimal monetary policy could be implemented in an econometric model of the Norwegian economy called the Norwegian Aggregate Model (NAM), see Bårdsen and Nymoen (2009a). They also analyzed how much the predicted economic outcome depended on the specification of the supply side of the model, in particular as WP-ECM or PCM.

In the analysis, they used a theoretically derived rule for setting the interest rate i_t, cf. Akram (2010):

$$i_{t+h} = i_0 + \beta_{\varepsilon,H}\varepsilon_t + g_H(i_{t+h-1} - i_0),\ h = 0, 1, 2..., H. \tag{31}$$

This rule defines an interest rate path corresponding to a specific policy horizon H. The response coefficient

$$\beta_{\varepsilon,H} = (1 - g_H)\frac{\beta_\varepsilon}{(1 - \phi_\pi)},$$

determines by how much the interest rate must deviate initially from the neutral rate i_0 to counteract inflationary effects of a shock ε_t. A high value of the smoothing parameter g_H can be associated with a strategy of gradualism in interest rate setting. Thus, the parameters $\beta_{\varepsilon,H}$ and g_H depend on the policy horizon, H. The last parameter in the theoretical interest rate equation is ϕ_π, which represents the (objective) degree of persistence in the inflation shock.

In addition to the preferences about policy horizon and gradualism captured by (31), the optimal interest rate response is influenced by the user's choice of macroeconomic model, which can be thought of as determining β_ε. Akram and Nymoen (2009) makes use of three versions of an empirical macro model of the Norwegian economy, cf. Bårdsen and Nymoen (2009a). In one model version, nominal wage and price setting is modelled by the WP-ECM framework above. The two other versions are: The model with wage and price Phillips curves, PCM, , and a version with a vertical Phillips curve, not unlike the restricted PCM in the analysis of U.S. data mentioned before.

Akram and Nymoen showed that econometric encompassing tests favoured the WP-ECM model of the supply side, and the ensuing analysis suggested that the econometric test result contained valuable information of practical economic significance.

By the use of model simulation, the authors identified a trade-off between price and output stabilization for different ranges of policy horizons. Specifically, in the case of WP-PCM and PCM the trade-off was in the range of 0 to 8 quarters. Policy horizons that are longer than 8 quarters appeared inefficient as both price and output stabilization could be improved by shortening the policy horizon. The opposite was the case for the restricted PCM. In that case, the trade-off curve was associated with policy horizons that were longer than 6 quarters, while policy horizons shorter than 6 were inefficient. Even though the efficiency frontiers for WP-ECM and PCM were defined by almost the same policy horizon, the optimal horizon (for the given loss function) was found to be three quarters conditional on WP-ECM, but six quarters in the case of PCM. In the case of the restricted PCM the policy horizon was eleven quarters.

Based on the above and other simulation experiments, Akram and Nymoen found that econometric differences bear heavily on model-based policy recommendations and thus were not merely of academic interest. Hence, monetary policy based on a mis-specified model of wage formation may lead to substantial losses in terms of economic performance, even when policy is guided by gradualism, for example in the form of a long policy horizon.

Wage formation and business cycle "stylized facts"

Using empirical WP-ECMs for the United Kingdom and Norway, Bårdsen et al. (1998) analyzed the phenomenon of empirically unstable price-wage output correlations (r_{xy}) which had been noted as a puzzle in business cycle analysis during the 1990s, cf. Blackburn and Ravn (1992), Englund et al. (1992), Andersen (1994, p 18-19) among others. They demonstrated that the econometric models could resolve the puzzle of unstable r_{xy} values. A main conclusion was that only when shocks (impulses) come from the same part of the macroeconomic system (e.g., the demand side) can the pairwise correlations be expected to be stable for different samples, or to be consistent in cross-country comparison. Conversely, significant breaks in r_{xy} values do not logically imply a structural change in wage and price setting.

Consequently, care must be taken in the usage of pairwise correlations in the calibration of theory-driven models, for example real-business-cycle models and DSGE models. Indeed, as pointed out by Hendry (1995b), the practice of matching subsets of moments that are inherently non-constant induces a sample dependency in models that were intended to avoid exactly this difficulty.

Norwegian wage formation since industrialization

Nymoen (2017) modelled the Norwegian wage level over a long historical period, from 1900 to 2015. Norway was industrialized relatively late, and the data therefore covers most of its economic and institutional development since industrialization. An empirical WP-ECM from the paper is shown in compact form in display (32). The endogenous variables are w, q, u and p. The exogenous variables are a, pi and four variables that capture *breaks* in wage and price setting and unemployment. In the same way as in the theory above, a and pi are modelled as random walks with drift (results not shown).

$$
\begin{pmatrix} 1 & 0 & 0.03 & -0.5 \\ -0.36 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.98 \\ 0 & 0.33 & 0 & 1 \end{pmatrix}
\begin{pmatrix} \Delta\widehat{w}_t \\ \Delta\widehat{q}_t \\ \Delta\widehat{u}_t \\ \Delta\widehat{p}_t \end{pmatrix}
=
\begin{pmatrix} 0.24 & 0 & 0.23 & 0.13 & 0 \\ -0.36 & 0.34 & 0.1 & 0 & 0 \\ 0 & -0.98 & 0 & 0 & 0.3 \\ 0 & 0.16 & 0.31 & 0 & 0 \end{pmatrix}
\begin{pmatrix} \Delta a_t \\ \Delta pi_t \\ \Delta p_{t-1} \\ \Delta wc_{t-1} \\ \Delta u_{t-1} \end{pmatrix}
$$

$$
+\begin{pmatrix} -0.08 & -0.01 & 0 & 0 & 0 \\ 0 & 0 & -0.14 & 0 & 0 \\ 0 & -0.37 & 0 & -0.27 & -0.24 \\ 0.04 & 0 & 0 & -0.02 & 0.03 \end{pmatrix}
\begin{pmatrix} (wc-a-q)_{t-1} \\ u_{t-1} \\ q_{t-1} - 0.7(w-a)_{t-1} - 0.3pi_{t-1} \\ (w-p-a)_{t-1} \\ (pi-p)_{t-1} \end{pmatrix}
$$

$$
+\begin{pmatrix} 0.91 & 0 & 0 & 0 \\ 0 & 0.96 & & 0 \\ 0 & & 0.95 & \\ & & & 1.1 \end{pmatrix}
\begin{pmatrix} wbreak_t \\ qbreak_t \\ ubreak_t \\ pbreak_t \end{pmatrix}
\tag{32}
$$

The estimation method is full information maximum likelihood (FIML). To save space, the constant terms and the payroll-tax variable, which only enters the wage equation, have been omitted. For the same reason, standard errors of the coefficient estimates are not shown. But all individual 't-tests' would be significantly different from zero at the usual significance level.

The model (32) is an encompassing model, in the meaning that the restrictions it implies for the unrestricted VAR (reduced form) are jointly statistically insignificant. When the set of coefficient restrictions which defines the PCM is imposed, the null hypothesis of PCM as the valid model was statistically rejected. However, as above, a relevant question is whether the set of restrictions makes for numerically significant differences between the models' properties.

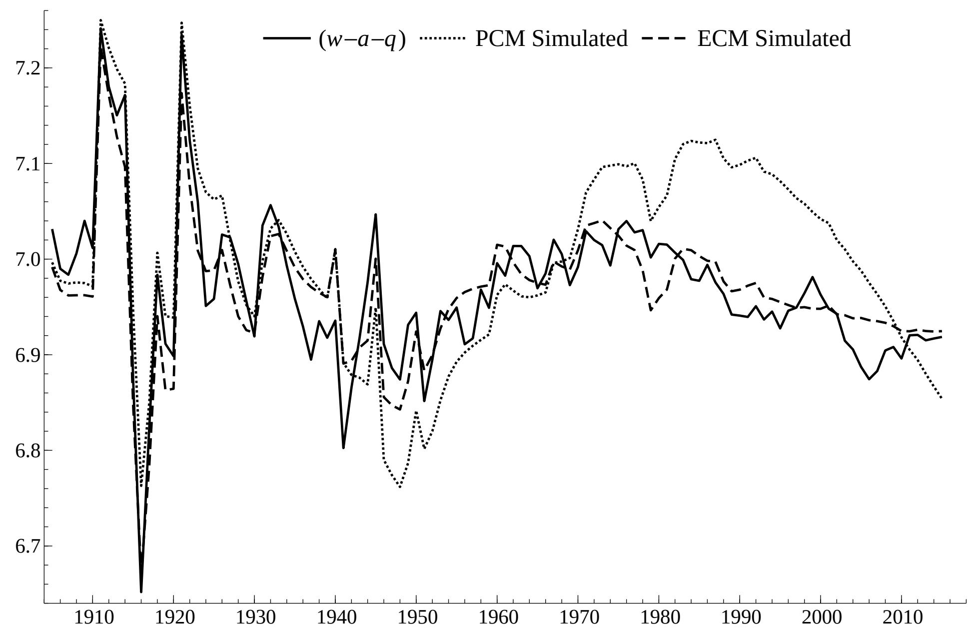

Figure 4: Dynamics simulation of the wage share

Figure 4 gives some indication of such numerical significance for the wage share. The solutions are obtained by dynamic solution of WP-ECM and the PCM versions of the model, with 1905 as the first solution period. The solutions appear to be almost identical for the first decades or so, but they gradually diverge, with the PCM solution the least "I(0)"-like. This indicates that in this case, the extended equilibrium property is not very strong. Hence in this example there is a significant difference in the solutions of the PCM and ECM, even before the hypothesis of a vertical long-run Phillips curve is imposed, which would be a counterpart to the restricted PCM model in the model for U.S. postwar wage and price inflation.

Forecasting

As noted by e.g., Nymoen (2019, Ch. 12), there is a common ground between forecasting methods that are often seen as alternative and competing, such as univariate time series models and larger systems of equations. For example, the forecasting function of a variable in a dynamic system, small or large, has a *glide path* interpretation. Its origin is an observed (or "nowcasted") starting point, and the end point of the glide path is the estimated long-run mean (*ie* mathematical expectation of the variable). The glide path of linear models is continuous between the starting point and the end point, but it is not monotonous in general. If the variable being forecasted is $I(1)$ non-stationary, only the starting point is well defined. However a cointegrating linear combination of $I(1)$-variables follows a well-defined glide path when forecasted.

The forecasting properties of PCM and WP-ECM can be analyzed as special cases. Let the known initial values be given by the information set $\mathcal{I}_T^m$, where $m = PCM$ or $m = WP\text{-}ECM$ and assume that the model forecasts are conditional expectations based on $\mathcal{I}_T^m$. It then follows that for each endogenous variable, Y_i, all dynamic forecasts H periods ahead from T will be model dependent:

$$Y_{i,T+H|T}^{mf} = E(Y_{i,T+H} \mid \mathcal{I}_T^m). \tag{33}$$

As the forecast horizon H is extended toward infinity ("long-term forecasts"), the forecasts glide toward the unconditional expectations, which will however still be model dependent.

To show the glide-path interpretation of the PCM and the WP-ECM forecasts, it is convenient to reuse the notation from the model typology section for the closed economy case (hence $\Delta p_t = \Delta q_t$), only adding the constant terms (c_w and c_q):

$$\Delta w_t = c_w + \pi_w \Delta q_t + \tau_w \Delta a_t - \beta_w ws_{t-1} - \sigma_w u_t + \varepsilon_{wt} \tag{34}$$

$$\Delta q_t = c_q + \tau_p(\Delta w_t - \Delta a_t) + \beta_p ws_{t-1} + \varepsilon_{qt} \tag{35}$$

$$u_t = c_u + \varrho ws_{t-1} + \alpha u_{t-1} + \varepsilon_{ut} \tag{36}$$

In order to simplify, and without loss of generality given the assumption of correct model specification, we can abstract from estimations issues and assume that the coefficients the within sample coefficients are known. . The wage share and the unemployment rate are forecasted H periods ahead from T: ws_{T+h} and u_{T+h} $h = 1, 2, ..., H$. To simplify notation, let $\alpha = \tau_p = \tau_w = 0$ and let g_a denote the growth rate of productivity as before. This gives, for the wage share forecasts:

$$ws_{T+H}^{\text{WP-ECMf}} \underset{H\longrightarrow\infty}{\longrightarrow} E^{\text{WP-ECM}}[ws_t] = \frac{(c_w + (\pi_w - 1)c_q - \sigma_w c_u - g_a)}{-\{(\pi_w - 1)\beta_p - \beta_w - \sigma_w \varrho\}}, \tag{37}$$

$$ws_{T+H}^{\text{PCMf}} \underset{H\longrightarrow\infty}{\longrightarrow} E^{\text{PCM}}[ws_t] = \frac{(c_w + (\pi_w - 1)c_q - \sigma_w c_u - g_a)}{\sigma_w \varrho}. \tag{38}$$

Clearly, the end-points of the two forecasts are not identical. Since the unemployment forecasts obey the same equation (for the two models):

$$u_{T+H}^{mf} = c_u + \varrho ws_{T+H}^{mf} \tag{39}$$

the end-points of the unemployment forecasts will also be different: $u_{T+H}^{\text{WP-ECMf}} \neq u_{T+H}^{\text{PCMf}}$. For the PCM specifically:

$$u_{T+H}^{\text{PCMf}} \underset{H\longrightarrow\infty}{\longrightarrow} \frac{(c_w + (\pi_w - 1)c_q - g_a)}{\sigma_w} \tag{40}$$

i.e., the NAIRU rate above. In the case of $\pi_w = 1$:

$$u_{T+H}^{\text{PCMf}} \underset{H\longrightarrow\infty}{\longrightarrow} \frac{(c_w - g_a)}{\sigma_w}, \text{ natural rate } \ u^*. \tag{41}$$

It follows that in a stationary world with no structural breaks in the forecast period, only one set of model based forecast will be unbiased. The forecasts of the model which is the best approximation of the DGP. The forecasts of the other model of the supply side will have larger mean squared forecasted error (MSFE).

A main source of systematic forecast errors in macro models are structural breaks in the DGP that happen in the forecast period or close to (or in) period T. It is impossible to protect forecasts against the first type of break. The second type can be difficult to discover and assess in practice. In this situation it is not certain that the model that best approximates the DGP within sample also has the lowest MSFE. The literature has shown that models that only use differenced macro variables, so-called dVARs, may be more robust than the forecasts of a model that makes use of levels variables, that is, in the way that the WP-ECM does, see e.g., Clements and Hendry (1999).

An attempt to analyze the empirical forecast performance of WP-ECM and PCM was reported in (Bårdsen et al., 2005, Ch. 11). The analysis showed that although the PCM shared some of the robustness of dVARs, it also embodies equilibrium correction in the form of natural rate dynamics. Because that form of correction mechanism was rejected empirically, the PCM forecasts were harmed both by excessive uncertainty (from its dVAR aspect), and by their econometric mis-specification of the equilibrium-correction mechanism in wage formation.

The New Keynesian Phillips Curve (NKPCM)

A common feature of the WP-ECM and the PCM is that wage changes (wage inflation) are jointly determined with changes in the price level (inflation). The joint dependency is not necessarily simultaneous, but the models capture that one cannot have wage inflation without price inflation, and vice versa. Looking back at the typology (section), this is not the case for the New Keynesian Phillips curve model, (10)-(11). In that model, inflation is not explained by wage changes. The wage level (relative to the value of labour productivity) does, however, play a role, as the wage share is the forcing variable in the NKPCM inflation equation. Instead of an equation for nominal wage change, the NKPCM is specified with a first-order autoregressive process for the wage share (ws_t):

$$ws_t = (1 - \beta_{w3})ws_{t-1}$$

Hence, taken at face value the NKPCM is tacit about how the wage share and the nominal wage hang together with inflation and the other variables in the macroeconomic system. However one can reapply the same type of theory to wage formation, with some modifications. This was done by Galí (2011), who specified a *wage* New Keynesian Phillips curve. Taken together, the price NKPCM and the wage NKPCM define a model of the supply side that is comparable to the WP-ECM and PCM .

The New Keynesian Phillips curve

The best known version of the NKPC is attributed to Galí and Gertler (1999) and is defined as:

$$\Delta p_t = c_p + \underset{>0}{\pi_p^f} E_t[\Delta p_{t+1}] + \underset{\geq 0}{\pi_p^b} \Delta p_{t-1} + \underset{>0}{\beta_p} ws_t + \epsilon_{p,t}, \tag{42}$$

where the notation of the typology has been used, only augmented by the addition of an intercept and a random error term.[9] A hallmark of this theory is that the lead-term Δp_{t+1}^e in the typology is defined as $E_t[\Delta p_{t+1} \mid \mathcal{I}_t]$, that is, the expected rate of inflation in period $t+1$, given the information available for forecasting at the end of period t.

In Galí and Gertler (1999), and several others, there is no error term in (42) (implying $\epsilon_{p,t} = 0$).

[9] The "model 3" part of the notation has been dropped for simplicity.

It is custom to refer to it as the exact form of the structural equation, e.g., Sbordone (2002). The version with $\epsilon_{p,t}$ specified as for example, a Gaussian white noise process conditional on time t information, is referred to a the inexact form of the NKPC. The distinction between exact and inexact form becomes important for methods that test the restrictions that the rational expectations solution implies for the realistic case of persistence in both the inflation and the forcing variable. Fanelli's (2007,2008) early contributions that analyzed this point theoretically and empirically in a (cointegrated) VAR framework, see also the recent work by Boug et al. (2017) and the references therein.

Setting $\epsilon_{p,t} = 0$ is probably not meant as a claim about having the exact theory. However, it does reflect a line of thinking saying that the empirical verification of the properties of a random errorterm in the structural equation is relatively unimportant, for example because robust estimation methods like GMM are in any case used to estimate the coefficients. However there is no logic saying that implicit, imprecise or "weak" probabilistic assumptions render the conclusions of an empirical study (quantification) less vulnerable to statistical mis-specification. When the purpose is statistical induction, the premium is on testability of assumptions. When validated, stronger assumptions may give a much more effective way of learning from the data than weaker assumptions do Spanos (2021). The decision to specify the model with or without an random error term also has implications for system (VAR) based tests of the new Phillips curve theory.

In the approach followed here, the "inexact form" of the structural inflation equation is used without complicating the derivation, estimation or empirical evaluation. The econometric inflation equation thus obtained is accompanied by the specification of the process for the forcing variable ws_t as a second-order autoregressive process, thus generalizing the notation in the typology in a way that conforms with many applications:[10]

$$ws_t = c_{s0} + c_{s1} ws_{t-1} + c_{s2} ws_{t-2} + \varepsilon_{s,t}, \tag{43}$$

The solution for Δp_t, using (42) and (43), becomes:

$$\Delta p_t = b_{p0} + b_{p1} \Delta p_{t-1} + b_{p2} ws_t + b_{p3} ws_{t-1} + \varepsilon_{p,t}. \tag{44}$$

[10]In the first order case, $c_{w1} = 1 - \beta_{w3}$ in the notation of the model typology

This equation is an autoregressive distributed lag (ADL) model equation, well known from dynamic econometrics. Given the assumptions of the theory, ws_t is uncorrelated with the error term $\varepsilon_{p,t}$ which is proportional to the error term in (42).[11] Hence, if it is assumed that the structural error-term $\epsilon_{p,t}$ is Gaussian white-noise, $\varepsilon_{p,t}$ in (44) is also normally distributed. Hence, OLS estimation of (44) gives approximate maximum likelihood estimators which are consistent. Formal (by encompassing tests) or informal comparison with competing specifications is therefore straightforward to do, cf. Nymoen (2019, Ch. 7.11).

In the literature that has critically assessed the New Keynesian theory of inflation, this route remains largely unexplored. Instead researchers have focused on IV and GMM estimation of the structural equation (42), see Rudd and Whelan (2005), Bårdsen et al. (2004), Castle et al. (2013) and Mavroeidis et al. (2014) which overviews the results for the π^f coefficient in particular. After the Great Recession, more evidence against the structural form of has accrued, Ball and Mazumder (2019). However, when Galí (2011) introduced the New Keynesian *wage* Phillips curve (see below) he proposed to estimate a solution equation similar to (44), as a way of obtaining empirical validation of his wage model.

Returning therefore to the solution in equation (44), it is worth noting that the coefficients of (44) are combinations of the parameters of (42) and (43) see e.g., Bårdsen et al. (2005, Appendix A) building on Pesaran (1987) and Nymoen et al. (2012). b_{p1} is identical to the stable root (r_1) of the associated characteristic equation of (42). The two distributed lag coefficients depend on the parameters of (42) and (43).

One final point is that the constant term b_{p0} in (44) is also a confluent term. Setting $c_p = 0$ without loss of generality, it can be shown that (cf. Nymoen (2014)):

$$b_{p0} = \frac{\beta_p}{\pi_p^f}(\frac{1}{r_2 - 1})\mu_s, \tag{45}$$

where $r_2 > 1$ is the so-called unstable root and μ_{ws} is the expectation of the wage share ws_t, that is:

$$\mu_s = \frac{c_{s0}}{1 - c_{s1} - c_{s2})}. \tag{46}$$

Hence, if there is a change in the mean of the process of the forcing variable, there should theo-

[11] $\varepsilon_{p,t} = 1/(\pi^f r_2)\epsilon_{p,t}$, the scaling factor is $1/(\pi^f r_2)$ where $r_2 > 1$.

retically also be a change in the constant term of the solution equation of inflation, (44). Because structural breaks in the form of location shifts are common in economic time series, this represent an additional implication of the theory that is testable.

b) The New Keynesian Wage Phillips Curve

The wage-NKPC, due to Galí (2011) (with error-term added) is:

$$\Delta w_t = c_w + \underset{>0}{\pi_w^f} E_t[\Delta w_{t+1}] + \underset{\geq 0}{\pi_w^b} \Delta \bar{\pi}_{t-1} - \underset{>0}{\sigma_w} u_t + \epsilon_{w,t}, \tag{47}$$

where $\Delta \bar{\pi}_t$ is "the price inflation variable to which wages are indexed", Galí (2011, p 441).[12]

The forcing variable in (47) is the unemployment rate, with generating equation:

$$u_t = c_{u0} + c_{u1} u_{t-1} + c_{u2} u_{t-2} + \varepsilon_{u,t}, \tag{48}$$

where $\varepsilon_{u,t}$ can be assumed to be Gaussian white noise without loss of generality. The solution for Δw_t given (47) and (48) is:

$$\Delta w_t = b_{w0} + b_{w1} \Delta \bar{\pi}_{t-1} + b_{w2} u_t + b_{w3} u_{t-1} + \varepsilon_{w,t}. \tag{49}$$

In the same way as for (44), the coefficients of (49) depend on the parameters of (47) and (48). In particular the constant term b_{w0} contains the expectation of u_t. Hence, if there is a structural break in for example c_{u0}, this theory predicts that also b_{w0} should break.

However, the point made by Galí (2011) was more in the direction of seing (49) as a reduced form wage inflation that rested on better theory than given hitherto for the typical US wage Phillips curve.[13]

A possible objection to the suggestion to use (49) for econometric evaluation of New Keynesian theory is that it is based on the exogenous AR(2) process (48). After all, basic macroeconomic theory implies that wage inflation can be expected to be Granger-caused by unemployment, and the DSGE model of which wage NKPCM is meant to be part, is no different in this regard. Hence

[12] To save notation, the mean of u_t is subsumed in c_w. Equation (14) in the paper also includes the term $E_t[\bar{\pi}_t] = \bar{\pi}_t$, which is however omitted from the solution expression (18) in the paper, corresponding to (49).

[13] Galí allows for an autocorrelated error-term in his estimated equation, due to "measurement error in the wage inflation data" (p.452) and not to potential mis-specification. However, Table 1, only reports OLS results and there are no diagnostic tests of residual mis-specification.

it may seem self-contradictory to assume an exogenous unemployment process as a step in the derivation of the solution (49). One way around this is Campbell and Schiller's (1987) method for assessment of present value relations, which however requires the undesirable assumption that (47) holds exactly.

However, it is standard in the New Keynesian DSGE model literature to assume that the data generating process (DGP) belongs to the unique stable solution of the multiple-equation model that allows for mutual feedback. Under this assumption, the DGP takes the form of a globally asymptotically stationary VAR, see Bårdsen and Fanelli (2015). There are implied *final equations* for every endogenous variable of the VAR. The final equations are difference equations that have identical homogenous parts, whereas the non-homogenous parts are (by definition) heterogenous between variables, see e.g., Nymoen (2019, p. 142), Hendry (1995b, p. 340).

In this light, equation (48) can be interpreted as the final form equation of u_t, from a stationary VAR with first-order dynamics with white noise errors.[14] One caveat is that a VAR with white noise errors has final equations which has moving-average errors, that is, the non-homogenous parts are composites of the VAR errors. Hence, strictly speaking if (48) is interpreted as a final form equation, $\varepsilon_{u,t}$ is a moving-average process. Because that property of (48) was not taken into account in the derivation of the solution equation (49), the result mentioned above about $\varepsilon_{w,t}$ being a white-noise error term may need to be modified.

However, it has been shown that a moderately over-parameterized AR model gives a close approximation of an ARMA model, that is, in terms of fit and with respect to the linear properties of the time series, see Wahlberg (1989), Hendry (1995a, Ch 15.7) and the references therein. Consequently, proceeding from the assumption that (48), with $\varepsilon_{u,t}$ as white noise, is defensible as an approximation to a first-order ARMA model of u_t, the cost of the approximation is perhaps some loss of parsimony.

Hence a feasible method is to include all significant lags in the final form equation of u_t, subject to the residuals being empirically white noise. The lag order can then be set to for example three, instead of two as in (48). The extended dynamics in the final form equation creates no problems for the derivation of the solution equation for Δw_t. The consequence will be that one or more

[14]Interestingly, the VAR assumption is also made by Galí (2011) in order to construct a time series for so called fundamental wage inflation by using Campbell and Schiller's procedure.

additional lags in u_t is included in the equation. Hence, if the final form equation is of order three, there will be a distributed lag of order two in u_t in the wage change equation, instead of the first order distributed lag in (49).

A similar remark can be made for the price New Keynesian price Phillips curve (44), which can be augmented with longer lags in the ws_t if that helps in the specification of a non-mis-specified model equation for price level changes.

Examples of empirical New Keynesian solution equations

Collecting equations a "NKPCM-system" becomes:

$$\Delta p_t = b_{p0} + b_{p1}\Delta p_{t-1} + b_{p2} ws_t + b_{p3} ws_{t-1} + \varepsilon_{p,t}, \tag{50}$$

$$\Delta w_t = b_{w0} + b_{w1}\Delta \bar{\pi}_{t-1} + b_{w3} u_t + b_{w3} u_{t-1} + \varepsilon_{w,t}, \tag{51}$$

$$ws_t = \Delta w_t - \Delta p_t - \Delta a_t + ws_{t-1} \tag{52}$$

$$\bar{\pi}_t = 0.25\Delta_4 p_t \tag{53}$$

where the two last equations are identities that help close this New Keynesian model of the supply side. In (53) the wage-indexation term $\bar{\pi}_t$ is specified as the year-on-year price inflation, which was used by Galí (2011). [15]

Hence, the equations of the NKPC system can be evaluated as empirical models, by the use of standard methods of time series econometrics. As noted, based on the underlying theory, the two error terms are white noise and homoskedastic. Nevertheless Galí (2011) used robust standard error in tables with estimation results, with reference to measurement errors in the wage data. However, there are several other sources of mis-specification, and Spanos (2021) reminded us, there is very little robustness without statistical adequacy.

As an illustration, using the same data and sample period (1960(1)-1997(4)) as Galí and Gertler (1999), the estimated version of (50) becomes:

$$\Delta p_t = \underset{(0.000447)}{0.00122} + \underset{(0.0398)}{0.8732}\ \Delta p_{t-1} + \underset{(0.028)}{0.06879}\ \Delta ws_t + \underset{(0.0154)}{0.0227}\ ws_{t-1} \tag{54}$$

$$\hat{\sigma}100 = 0.27 \quad R^2 = 0.812 \quad T = 152$$

[15] cf. Galí (2011, Table 1).

Given the theoretical preeminence of the wage share as the forcing variable in the NKPCM, a puzzling result in this equation is the statistical insignificance of the coefficient of ws_{t-1} (i.e., $(b_{p2} + b_{p3})$): t-value 1.5. The relative weakness of the wage share as a forcing variable has been pointed out in the literature, implying that there is little "inherited" persistence from the expectation formation about forcing variables, and mainly "intrinsic" inflation persistence, Fuhrer (2006).

Standard mis-specification tests for residual autocorrelation and heteroskedasticity are significant when they are applied to (54).[16] The underlying New Keynesian theory allows inclusion of more lags in the forcing variable, if that improves the model specification. However, adding three lags draws a blank as the test of joint significance becomes: $F(3, 145) = 1.34[0.26]$.

Finally, note that another implication, namely that structural breaks in the wage share process should induce breaks in the solution equation of Δp_t, can also be investigated. A feasible approach is to use the impulse indicator saturation (IIS) method which is a part of the automatic variable selection algorithm Autometrics, Doornik (2009), Hendry and Doornik (2014). Estimating the final form equation (43) of ws_t using IIS with overall significance level 1 % Autometrics returns 11 indicator variables. When this set of indicators for structural breaks in the mean of inflation was added to (54) only two dummies had significant t-values: the dummies for 1977(4) and for 1978(2). Conversely, application of Autometrics-IIS to (54) gave 12 indicator variables, but only one of them, for 1977(4), was also a break in the equation of the forcing variable ws.

The probability of finding spurious breaks can be controlled by choosing a lower overall significance level in the algorithm. Repeating the above estimation with significance level 0.1 %, Autometrics finds no breaks in the ws equation, but returns three break dummies for (54): 1972(2), 1974(3) and 1977(4). In summary there is less "synchronization" between the breaks in the Δp_t equation and in the process of the forcing variable than the New Keynesian theory predicts.

The estimated version of the wage NKPCM (51) shown in (55) fares better in terms of the strength of the forcing variable. The sum of the two unemployment coefficients is statistically significant, and is quite sizeable. Of course, this is not surprising considering the empirical success of the wage PCM when estimated on U.S. data.

$$\Delta w_t = \underset{(0.0016)}{0.0128} + \underset{(0.07435}{0.945149} \ \Delta \bar{p}_{t-1} - \underset{(0.1234)}{0.179521} \ \Delta u_t - \underset{(0.02779)}{0.142321} \ u_{t-1} \tag{55}$$

[16] Applying robust standard error reduces the implied t-value of the lagged wage share a little more, to 1.3.

$$\hat{\sigma}100 = 0.467 \quad R^2 = 0.549 \quad T = 152$$

Using a 1 % significance level, Autometrics-IIS finds eight break indicators in the final form equation of u_t, however only one of them, 1960(1), is statistically significant when added to the New Keynesian wage Phillips curve (55). In the same way as for the price NKPC, this indicated less pass-through than theory predicts, of breaks in the mean of the forcing variable to the wage equation. This does not however imply that the price NKPC has constant mean. With the 1 % level of overall significance, Autometrics-IIS finds 8 break dummies in this equation as well, but only one of them, 1960(1), is found to be a structural break also in the wage equation.

Tests based on the structural equation and encompassing tests

The price and wage versions of the New Keynesian Phillips curve are examples of economic models that include expected future values to explain current outcomes. As noted above, models of this type are estimated by IV or GMM after replacing the expected value by the actual future outcome. The focus of this literature has been on the empirical evidence on inflations expectations in the (price) New Keynesian Phillips curve. Mavroeidis et al. (2014) is a comprehensive survey.

One feasible way of testing the structural equation more broadly is by running encompassing tests against other relevant models of wage and price setting. Specifically, the following procedure can be followed:

1. Assume that there exists a set of variables $z = [z_1, z_2]$ where the subset z_1 is sufficient for identification of the structural NKPCM model equation.

2. Using the identifying subset z_1 , estimate by IV (or by GMM) an augmented NKPCM where the augmentation consists of the other sub-set of variables, z_2.

3. Under the hypothesis that the NKPCM is the correct model, the coefficient vector of z_2 in the augmented NKPCM equation is zero, and the numerical and statistical significance of the lead variables should be more or less be that same as found in 1.

In addition to inspection of the t-values of the relevant variables, one can also estimate the over-identified NKPC (all z-variables are used as instruments), and interpret the Sargan (1958),1964) specification test, Davidson and MacKinnon (2004, 8.6), or the equivalent J-test Hansen (1982), as encompassing tests. These test statistics should be insignificant if the NKPCM is the correct model equation.

In terms of practical implementation one could take advantage of existing results on wage and price modelling using cointegration analysis which readily delivers z_2-variables in the form of linear combinations of levels variables. In other words, they represent "unused" identifying instruments that go beyond information sets used in the estimation of structural NKPCM. An example is given by Bårdsen et al. (2005, Ch. 7.5), who used instrumental variables from the study by Bårdsen et al. (1998) cited above to test the encompassing properties of an empirical NKPC model equation for the U.K. economy, specified by Batini et al. (2005).

Of course, for encompassing tests to affect beliefs about wage and price setting, there must be some common ground, in the form of mutual recognition of relevance of empirical findings, and of the results of preexisting studies. It seems that sometimes the pre-eminence of theory in the specification of empirical econometric models has intervened in a way that have reduced the role of encompassing testing in the progress of the discipline.

An alternative approach, which does not require variables from other studies to be introduced, but which also uses the equations with explicit lead terms, utilizes the implicit parameter invariance assumption the NKPCM model. The point is that since there is no allowance for unanticipated regimes shifts in the reduced forms of the lead variables in either (42) or (47), a significant and sizable IV estimated coefficient of Δp_{t+1} or Δw_{t+1} can potentially be spurious. Castle et al. (2013) proposed using a two-stage procedure. The first stage applies Autometrics-IIS to the reduced form of Δp_{t+1} to detect the presence of any unmodelled outliers or location shifts; and the second is to test for their presence in the structural equation (42). When the method was applied to both U.S and Euro-area price NKPCs, Galí and Gertler (1999) and Gali et al. (2001) the previous results became radically altered.

These methods can be regarded as complementary to the others ways of testing the NKPCM. The suggested new test that combine the price and wage NKPCM, does not require identifying assumptions for estimation, or over-identifying instruments for testing. It utilizes the implication that if the theory model is correct, a break in the mean of the forcing variable should go together with a break in the constant term of the solution equation. As structural breaks are known to be frequent in macroeconomics, the test should have some practical interest.

Concluding remarks

This article has offered a short and incomplete history of wage formation models summarized in three main conceptualizations: Phillips Curve models (PCMs), wage price equilibrium correction models (WP-ECMs) and New Keynesian Phillips curve models (NKPCMs). They make out a typology of candidate models of the supply side of medium-term macroeconometric models. The article continued by presenting the salient features of PCMs and WP-ECMs. Some of these features are shared between the models (e.g. dynamic joint causation), whereas other properties and implications are markedly different, for example, the definition and role of the steady-state rates of wage inflation and of unemployment. PCMs and WP-ECMs remain to be powerful conceptualizations of some of the main debates about how national macroeconomic systems function, and are therefore associated with competing beliefs about which policy strategies and measures are effective and ultimately also about which targets of economic policies that are feasible to pursue.

The common statistical model used for presentation of PCMs and WP-ECMs is the integrated VAR with a causal solution (i.e., only known initial conditions required). The difference between the PCM and WP-ECM can to some extent be brought down to specified assumptions about unit roots, cointegration and exogenity (which variables take the main roles in the adjustment of disequilibrium).

NKPCMs at first sight cannot be placed in the framework of a VAR with causal solution. This is due to its main definitional characteristic, the lead term in inflation (wage change in the case of the wage NKPCM). However, when the lead variable is modelled by the hypothesis of rational expectations, its solution can be assumed to have stationary and causal VAR representations. Exactly this approach was used to show that the implied solution equations of the hybrid price and wage NKPC take the form of "ordinary" dynamic regression models for wages and price, that can be compared (tested against) WP-ECM and PCM models.

The econometric treatment of wage formation for inclusion in macroeconometric models began during an era of full employment and relative stable prices in the Western postwar economies. The policy issue was then whether near full employment could be maintained without creeping inflation evolving into wage-price spirals. As inflation became recognized as endemic, the focus shifted in the direction of analyzing the role of wage formation and labour market institutions in the determination of the equilibrium unemployment rate, understood as the rate of unemployment that was reconcilable with stable inflation. More recently, but already two decades ago, the "inflation problem" quite suddenly disappeared in many modern economies. That change may have had less to with inflation targeting central bank policies than many believe, and more with China taking over the role as the workshop of the world, as well as other changes toward globalization going bach even longer in time.

During these epochs and periods of transition, the supply side of macroeconometrics models has evolved and has been modified. The changes can be interpreted as answers to the needs expressed by models users. However, there are also examples of direct influence from academic economics. The rise of the NKPCM, as part of the wider DSGE model movement, perhaps stands out as the main example of a theoretically driven change- However, earlier in the history of wage modelling, WP-ECM models were strongly influenced by the introduction of formal bargaining theory to economics, and the coming together of dynamic econometrics modelling and VARs (through cointegration methods).

The popularity of macro models tend to sink with forecast failures after major unanticipated location shifts. The oil crisis and stagflation years during the 1970s led to the rejection of large scale Keynesian models. DSGEs that failed in the Great Recession were however not discarded equally directly, which may be a sign of how deeply the emphasis on micro foundations had by then changed how academic macroeconomics was done, Wren-Lewis (2015). However, some central banks never switched to making DSGEs their main tool, even when the preeminence of theory of modern academic macro weighted in most heavily, and others have since developed non-DSGE models, Hendry (2020).

The different models of wage modelling have been integral to the macroeconometric modelling programs that have come and gone. This is likely to continue in the future. In the las two decades it has become normal to expect that below-target inflation and low interest rates will last "forever". During an epoch of low and stable inflation it is all too easy to lose sight of the complex dynamics in wage and price setting that is latent in modern economies. However, if not exactly on the horizon, inflation could be on the way back, Goodhart and Pradhan (2020). In that case, although a future inflationary epoch will be different from the historical ones, the advances already gained in empirical econometric modelling in this field will be a solid basis for the necessary revisions of knowledge, skills and understanding overall.

Further reading

A book that presented the wage-price model of the supply side as the pivot of an operational macroeconometric model was Bårdsen et al. (2005). That book also discussed the relationship between PCM, WP-ECM and NKPCM and the "Scandinavian model of inflation" that has had a large influence on the thinking that still underlies the systems of national wage setting in Norway (in particular) but also Sweden. An important contribution that gives a plausible explanation of why macroeconomic model builders first took a cautious approach to wage setting is Forder (2014). That book also drew an insightful distinction between the "Curve of Phillips" and the Phillips curve which quickly became a household model in the 1960s, referred to as PCM in this article. An important aspect of wage formation which has not been covered in this article is pattern wage bargaining (wage leader and wage follower) and the phenomenon of two tier bargaining. An econometric model of pattern wage bargaining using cointegration methodology is found in Gjelsvik et al. (2018). There are several entrances to the literature on macroeconometric methodology. Granger (1990) remains an important reference, not least because of the appraisal of the important differences that will exist between model producers and model users, regarding different aspects of model specification, estimation (and calibration), policy relevance and forecasting ability. Two recent contributions that deserve to be widely read are Hendry (2018) and Spanos (2021). Boland (2014) is an insightful book on model building in economics which also covers the different guises of macroeconomic modelling.

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